HARVARD
GRADUATE SCHOOL of EDUCATION

Does the Learning of Mathematics Build Higher-Order Thinking Skills?

Evidence from neuroscience and psychology

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Why do we teach mathematics?

- For discussion later today
- One reason that will likely come up is:
- Learning mathematics builds higher-order thinking skills
- My talk is about this conjecture.

- **Does learning mathematics build higher-order thinking skills?**

Plan for my talk

1. 'Unpacking' of this conjecture about the connection between mathematics learning and higher-order thinking skills.
2. Consideration of evidence in support of or that challenges this conjecture.

- 30-35 minute talk, followed by 10-15 minutes of questions and discussion, followed by lunch!

Math ↦ Higher-order thinking?

- Widely held belief
- Has deep roots in Western thought
- "*Those who are by nature good at calculations are, as one might say, naturally sharp in every other study, and … those who are slow at it, if they are educated and exercised in this study, nevertheless improve and become sharper than they were.*" [Plato, *Republic*, Book VII, Grube translation, 1974, p. 178. From Stanic, 1986.]

What is higher-order thinking?

- Informally used synonymously with:
  - Problem solving
  - Critical thinking
  - Reasoning
  - Deductive thinking
  - Logical thinking
- While these are not all identical, I use “higher order thinking” to refer to any and all of these

Math ➡ Higher-order thinking?

- *Claim 1*: When one learns mathematics well, does one also develop a set of "extra mathematical competencies"?
- Extra mathematical competencies:
  - Knowledge that is above and beyond mathematical content
  - Critical thinking, problem solving, logical thinking, higher-order skills, etc.

Math ➡ Higher-order thinking?

- *Claim 2*: Is it possible to teach extra mathematical competencies, either implicitly and/or explicitly?
- Higher-order thinking skills may develop naturally from learning math.
- Such skills may also be the focus of explicit instruction or curricula in conjunction with the math.

Math ➡ Higher-order thinking?

- *Claim 3*: Once learned, can extra mathematical competencies be applied or used in other settings?
- Other settings includes other domains in mathematics, other fields of knowledge where math may be useful, and mathematical or non-mathematical situations in everyday life.

Math ➡ Higher-order thinking?

- *Claim 1*: When one learns mathematics well, does one also develop a set of "extra mathematical competencies"?
- *Claim 2*: Is it possible to teach extra mathematical competencies, either implicitly and/or explicitly?
- *Claim 3*: Once learned, can extra mathematical competencies be applied or used in other settings?

Supporting evidence?

- Neuroscience?
  - No evidence at present in support of (or refuting) any of these claims
- Cognitive psychology
  - A great deal of evidence, from the past 100+ years
  - Claim 1: Theories of “mental discipline”
  - Claim 2: Teaching of critical thinking skills
  - Claim 3: Transfer

Claim 1: When one learns mathematics well, does one also develop a set of "extra mathematical competencies"?

- Theory of "Mental discipline"
  - Also known as formal discipline
- Widely held for centuries, up to the present
- The mind is like a muscle
- As muscles need exercise, so does the mind
  - From rigorous study, where the content being studied is relatively unimportant
- Greek, Latin, and mathematics especially key

Claim 1: When one learns mathematics well, does one also develop a set of "extra mathematical competencies"?

- Beginning in late 19th and early 20th centuries, psychologists tried to confirm or refute the theory of mental discipline
- Numerous studies were conducted
- Psychologists became convinced that this theory was invalid

Claim 1: When one learns mathematics well, does one also develop a set of "extra mathematical competencies"?

- Mathematicians and math educators were (and for many, continue to be) unconvinced
- Why?

1. Perception that studies are flawed
2. Intuitive strength of this belief
3. Perception that psychologists were against the teaching of math generally
4. Perception that, with good enough teaching, anything is possible

Math ➡ Higher-order thinking?

- *Claim 1*: When one learns mathematics well, does one also develop a set of "extra mathematical competencies"?
- *The verdict*: Research on the theory of mental discipline raises serious questions about whether this claim is valid. Yet many in mathematics and mathematics education continue to cling to this belief.

Claim 2: Is it possible to teach extra mathematical competencies, either implicitly and/or explicitly?

- Lots of research on critical thinking and how it can be developed
- Students generally do not develop critical thinking skills spontaneously in school
- To promote critical thinking, special courses and curricula were needed
- Such courses supplement regular instruction

Claim 2: Is it possible to teach extra mathematical competencies, either implicitly and/or explicitly?

- Do these courses work?
- Students show improvement primarily on tasks that are very similar to those in the curriculum
- Critical thinking is very hard to teach

Math ➡ Higher-order thinking?

- *Claim 2*: Is it possible to teach extra mathematical competencies, either implicitly and/or explicitly?
- *The verdict*: Critical thinking abilities typically do not develop as a natural by-product of whatever content is learned. In addition, even when critical thinking is an explicit instructional focus, it is still very difficult to foster the development of critical thinking skills.

Claim 3: Once learned, can extra mathematical competencies be applied or used in other settings?

- "Transfer" – the ability to apply what one has learned in one context to another, different context
- "Low road" vs. "high road" transfer

Claim 3: Once learned, can extra mathematical competencies be applied or used in other settings?

- "Low road" transfer – "spontaneous, automatic transfer of highly practiced skills, with little need for reflective thinking"
- "Near" transfer
- Happens frequently
- With sufficient (and the right kind of) practice, mathematics skills can be transferred to another mathematical domain
- Yet many students struggle

Claim 3: Once learned, can extra mathematical competencies be applied or used in other settings?

- "High-road" transfer – "Deliberate, usually metacognitively guided and effortful decontextualization of a principle, strategy, or procedure"
- "Far" transfer or "mindful abstraction"
- Very difficult to find in laboratory studies
- What we learn is tied to the context in which it was learned

Math ➡ Higher-order thinking?

- *Claim 3*: Once learned, can extra mathematical competencies be applied or used in other settings?
- *The verdict*: Achieving transfer is extremely difficult, requiring a particular set of learning conditions that are difficult to produce. Spontaneous transfer of extra mathematical competencies is rare.

Math ➡ Higher-order thinking?

- *Claim 1*: When one learns mathematics well, does one also develop a set of "extra mathematical competencies"?
- *Claim 2*: Is it possible to teach extra mathematical competencies, either implicitly and/or explicitly?
- *Claim 3*: Once learned, can extra mathematical competencies be applied or used in other settings?

Conclusions?

- Does the learning of mathematics build higher-order thinking skills?
- There is very little evidence to indicate that this is the case.
- When we consider reasons why we teach mathematics, if you feel inclined to suggest that we teach mathematics because it builds higher-order thinking skills, I have not found evidence to support this connection.

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Thanks! Questions? Comments?

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