The History of Mathematics Teaching
The Tension Between Practice and Theory

Victor Katz

Joseph W. Dauben
Department of History
Herbert H. Lehmman College
and
Ph.D. Program in History
The Graduate Center
City University of New York
GEORG CANTOR
His Mathematics and Philosophy of the Infinite

Joseph Warren Dauben
The History of Mathematics Teaching
The Tension Between Practice and Theory

Victor Katz

Joseph W. Dauben
Department of History
Herbert H. Lehman College
and
Ph.D. Program in History
The Graduate Center
City University of New York
九章算術 Jiu zhang suan shu
Nine Chapters on the Art of Mathematics
Southern Song dynasty (printed 1213)
1983: 張家山漢簡 *Zhangjiashan Han jian* (Han Bamboo Slips from Zhangjiashan)

202–186 BCE

Excavated: 1983
Tomb No. 247, Zhangjiashan
Jiangling County, Hubei Province

算數書
*Suan Shu Shu*

*Wen Wu* (2000)
算数书
Suan Shu Shu
(A Book on Numbers and Computations)

Peng Hao, ed.
Title of the book: 算數書
Suan Shu Shu
(A Book on Numbers and Computations)

Suan Shu Shu, Peng Hao, ed.
2007: 嶽麓書院藏秦簡  *Yuelu shuyuan cang qinjian*
Qin Slips Collected by the Yuelu Academy
嶽麓書院

Founded in 976 in the 9th year of the Song Dynasty during the reign of Emperor 开宝 Kaibao

- Zhu Xi and
- Zhang Shi

Emperor Song Taizu 宋太祖
嶽麓書院藏秦簡《数》書釋文·簡注

嶽麓書院藏秦簡整理小組（執筆：肖燦）

說 明

《数》書經整理得到：
一，有編號的簡共二百三十余枚。
   殘片二十余片，無編號。
二，有完整算題75例。（現存題設條件和問題或答案能依據簡文列出算法式的算題就視為完整算題）。
   又有单独成題的“術”17例。
   又有記錄有谷物體積重量比率、兌換比率的簡32枚。
   未計入算題數目。
   又有記錄量制的簡3枚，未計入算題數目。

凡 例
一 釋文中異體字、假借字一般隨文注出，外加（ ）號。
二 簡文原有錯字，一般在釋文中隨注正字，外加（ ）號。原有脫字或衍文，釋文不加更動，
   在注釋中說明。
三 簡文原有殘缺字，可據殘筆或文例補足的，
   外加（ ）號。不能辨識的殘缺字，用 □ 表示，
   每字一格。其他符號：
   ……字跡模糊，字數不能確定。

1. 計算田地產量或租稅的算題
2. 計算土地面積的算題
3. 谷物換算
4. 衰分算題
5. 贏不足算題
6. 少廣算題
7. 計算體積的算題
8. 勾股算題
9. 營軍之術（此算題暫時未歸類）
10. 量制
嶽麓書院藏秦簡《數》書釋文·簡注

嶽麓書院藏秦簡整理小组（執筆：肖 燦）

說 明

《數》書經整理得到：
一，有編號的簡共二百三十余枚。
　　殘片二十余片，無編號。
二，有完整算題75例。（現存題設條件和問題或答案能依據簡文列出算法式的算題就視為完整算題）。
　　又有單獨成題的“術”17例。　　又有記錄有谷物體積重量比率、兌換比率的簡32枚。
　　未計入算題數目。
　　又有記錄量制的簡3枚，未計入算題數目。

凡 例
一 釋文中異體字、假借字一般隨文注出，外加（ ）號。
二 簡文原有錯字，一般在釋文中隨注正字，外加〈 〉號。原有脫字或衍文，釋文不加更動，
　　在注釋中說明。
三 簡文原有殘缺字，可據殘筆或文例補足的，
　　外加（ ）號。不能辨識的殘缺字，用 □ 表示，
　　每字一格。其他符號：
　　……字跡模糊，字數不能確定。
1. 計算田地產量或租稅的算題
2. 計算土地面積的算題
3. 谷物換算
4. 哀分算題
5. 嬴不足算題
6. 少廣算題
7. 計算體積的算題
8. 勾股算題
9. 營軍之術（此算題暫時未歸類）
10. 量制
嶽麓書院藏秦簡《數》書釋文·簡注

嶽麓書院藏秦簡整理小组（執筆：肖 燦）

說明

《數》書經整理得到：
一，有編號的簡共二百三十余枚。
　　殘片二十余片，無編號。
二，有完整算題75例。（現存題設條件和問題或答案能依據簡文列出算法式的算題就視為完整算題）。
　　又有單獨成題的“術”17例。
　　又有記錄有谷物體積重量比率、兌換比率的簡32枚。
　　未計入算題數目。
　　又有記錄量制的簡3枚，未計入算題數目。

凡例
一 釋文中異體字、假借字一般隨文注出，外加（ ）號。
二 簡文原有錯字，一般在釋文中隨注正字，外加（ ）號。原有脫字或衍文，釋文不加更動，
　　在注釋中說明。
三 簡文原有殘缺字，可據殘筆或文例補足的，
　　外加（ ）號。不能辨識的殘缺字，用□ 表示，
　　每字一格。其他符號：
　　……字跡模糊，字數不能確定。

1. 計算田地產量或租稅的算題
2. 計算土地面積的算題
3. 谷物換算
4. 壓分算題
5. 贏不足算題
6. 少廣算題
7. 計算體積的算題
8. **Gou-Gu Problems**
9. 營軍之術（此算題暫時未歸類）
10. 量制
口有圆材踝（埋）地，不智（知）小大，斲之，入材一寸而得平一尺，問材周大幾可（何）。即曰，半平得五寸，令相乘也，以深（0304）一寸为法，如法得一寸，有（又）以深益之，即材径也。（0457）
□有圓材薶（埋）地，不智（知）小大，甕之，入材一寸而得平一尺，問材周大幾可（何）。即曰，半平得五寸，令相乘也，以深 [0304]
一寸为法，如法得一寸，有（又）以深益之，即材徑也。 [0457]

Suppose there is a circular [piece of] wood buried in the ground, whose size is unknown, but cutting to a depth of 1 cun gives a chord of 1 chi, it is asked how great is the circumference of the [circular piece of] wood? [The method] says: half the chord is 5 cun, multiply it by itself and using the depth of 1 cun as the divisor, dividing gives the result in cun, again adding the depth [of the cut] gives the diameter of the wood.
□有圆材蕴（埋）地，不智（知）小大，斲之，入材一寸而得平一尺，問材周大幾可（何）。即曰，半平得五寸，令相乘也，以深 [0304]
一寸为法，如法得一寸，有（又）以深益之，即材径也。[0457]

Suppose there is a circular [piece of] wood buried in the ground, whose size is unknown, but cutting to a depth of 1 cun gives a chord of 1 chi, it is asked how great is the circumference of the [circular piece of] wood? [The method] says: halving the chord gives 5 cun, multiplying it by itself and using the depth of 1 cun as the divisor, dividing gives the result in cun, again adding the depth [of the cut] gives the diameter of the wood.
[JZSS 9.09]:
今有圆材埋在壁中，不知大小。以锯锯之，深一寸，锯道长一尺。问：径几何？
答曰：材径二尺六寸。
术曰：半锯道自乘，此术以锯道一尺为句，材径为弦，锯深一寸为股弦差之一半，锯道长是半也。
[JZSS 9.09]:
今有圆材埋在壁中，不知大小。以锯锯之，深一寸，锯道长一尺。问：径几何？
答曰：材径二尺六寸。
术曰：半锯道自乘，此术以锯道一尺为句，材径为弦，锯深一寸为股弦差之一半，锯道长是半也。
[JZSS 9.09]:
今有圆材埋在壁中，不知大小。以锯锯之，深一寸，锯道长一尺。问：径几何？
答曰：材径二尺六寸。
术曰：半锯道自乘，如深寸而一，以深寸增之，即材径。

Suppose there is a circular [piece of] wood imbedded in a wall, whose dimensions are unknown. If a saw cuts to a depth of 1 cun, the length of the cut is 1 chi long. The question: What is the diameter [of the piece of wood]? The answer says: The diameter of the timber is 2 chi 6 cun.

The method says: Multiply half of the length of the cut by itself, divide by the depth of 1 cun; adding the depth of 1 cun gives the diameter of the timber.
此术以锯道一尺为句，材径为弦，锯深一寸为股弦差之一半，锯道长是半也。

[Liu Hui Comments]: This [method] uses the length of the cut of 1 chi as the gou, and the diameter of the timber as the xian; the depth of the cut of 1 cun is one-half the difference between the gu and xian, and the length of the cut should also be halved.
此术以锯道一尺为句，材径为弦，锯深一寸为股弦差之一半，锯道长是半也。

[Liu Hui Comments]: This [method] uses the length of the cut of 1 chi as the gou, and the diameter of the timber as the xian; the depth of the cut of 1 cun is one-half the difference between the gu and xian, and the length of the cut should also be halved.
[ZJSS 9.06]:
今有池方一丈，葭生其中央，出水一尺。引葭赴岸，适与岸齐。问：水深、葭长各几何？
答曰：水深一丈二尺，葭长一丈三尺。
Suppose there is a [square] pond with a side of 1 *zhang*, in the middle of which a reed grows, extending 1 *chi* above the water. If the reed is pulled to the edge [of the pond], then it just reaches the edge. The question is: How much are both the depth of the water and the length of the reed? The answer says: The depth of the water is 1 *zhang 2 chi*. The length of the reed is 1 *zhang 3 chi*.
[ZJSS 9.06]:

术曰：半池方自乘，此以池方半之，得五尺为句，水深为股，葭长为弦。以句、弦见股，故令句自乘，先见矩幂也。以出水一尺自乘，减之，出水者，股弦差。减此差幂于矩幂则除之。余，倍出水除之，即得水深。差为矩幂之广，水深是股。令此幂得出水一尺为长，故为矩而得葭长也。加出水数，得葭长也。

臣淳风等谨按：此葭本出水一尺，既见水深，故加出水尺数而得葭长也。

The method says: Multiply half the side of the pond by itself.

[Liu Hui Comments]: This [general method] uses half the side of the pond, which gives 5 chi as the gou, the depth of the water serves as the gu, and the length of the reed is the xian. Using the gou and xian to find the gu, therefore let the gou be multiplied by itself and first find the area of the gnomon.
[ZJSS 9.06]:

术曰：半池方自乘，此以池方半之，得五尺为句，水深为股，葭长为弦。以句、弦见股，故令句自乘，先见矩幂也。以出水一尺自乘，减之，出水者，股弦差。减此差幂于矩幂则除之。余，倍出水除之，即得水深。差为矩幂之广，水深是股。令此幂得出水一尺为长，故为矩而得葭长也。加出水数，得葭长。

臣淳风等谨按：此葭本出水一尺，既见水深，故加出水尺数而得葭长也。

The method says: Multiply half the side of the pond by itself.

[Liu Hui Comments]: This [general method] uses half the side of the pond, which gives 5 chi as the gou, the depth of the water serves as the gu, and the length of the reed is the xian. Using the gou and xian to find the gu, therefore let the gou be multiplied by itself and first find the area of the gnomon.
九章算術

The problem of the reed in the middle of a pond.

Jiu zhang suan shu
The Gou-Gu theorem may be reformulated as
\[c^2 - b^2 = a^2\]

Here, the entire area is \(c^2\), the yellow area represents \(b^2\), the two red areas are \((c-b)(b)\) and the blue-green area is \((c-b)^2\).

The area of the gnomon is \(a^2\), i.e. \(2(c-b)(b) + (c-b)^2\).
We now follow the directions of the method to find the depth of the pond, i.e. multiply half the side of the pond by itself, this is the $a^2$.

Since $a^2 = 2(c-b)(b) + (c-b)^2$, the next step in the method subtracts the length of the reed above water, $(c-b)$, multiplied by itself, i.e. $a^2 - (c-b)^2$, which equals $2(c-b)(b)$. 

Since $a^2 = 2(c-b)(b) + (c-b)^2$, the next step in the method subtracts the length of the reed above water, $(c-b)$, multiplied by itself, i.e. $a^2 - (c-b)^2$, which equals $2(c-b)(b)$.
Next the method divides the remainder by twice the length of the reed above water, $2(c-b)$:

$$\frac{a^2 - (c-b)^2}{2(c-b)}$$

which equals $b$. 
Since \( \frac{a^2 - (c-b)^2}{2(c-b)} = b \), the method instructs as a final step to add the length of the reed above water, \((c-b)\), and therefore:

\[
\left\{ \frac{a^2 - (c-b)^2}{2(c-b)} \right\} + (c-b) = b + (c-b) = c.
\]

Clearly, the method does give the length of the reed, \(c\), and the procedure is given in general terms, along with the specific numerical details of the given problem.
We now translate this same method into dealing with the analogous case of the log buried in the ground or imbedded in a wall.
The only difference between the reed-in-the-pond problem and the log-in-the-wall is that the dimensions of the lengths in the latter are exactly half those of the former. But since this is the only difference, the method proceeds exactly as in the case of the reed-in-the-pond problem.
To find the diameter of the circle \( c \), the method begins as before by squaring the gou, \( a \), which is half the length of the cut i.e. \( a/2 \):

\[
\frac{a^2}{4} = 2\left(\frac{1}{4}\right)(c-b)(b) + \frac{1}{4}(c-b)^2,
\]

the next step in the method divides by the depth of the cut, \( \frac{1}{2}(c-b) \), and after simplifying: \( a^2 / 2(c-b) \), which equals \( b + \frac{1}{2}(c-b) \).
The final step, to find the diameter of the circle $c$, proceeds as the method instructs, by adding the depth of the cut:

$$[a^2 / 2(c-b)] + \frac{1}{2} (c-b) = b + \frac{1}{2} (c-b) + \frac{1}{2} (c-b) = c,$$

the diameter obtained in complete generality.
算數書
*Suan Shu Shu*
*(A Book on Numbers and Computations)*

Peng Hao, ed.
Carrying charcoal from a mountain, in 1 day it is possible to carry 7 dou of charcoal to a wagon; the next day, 1 dan of the gathered charcoal taken to the wagon is transported to a government post (官 guan). Now wishing to go to the government post, carrying charcoal from (the mountain) and transporting charcoal the long distance to the government post, the question is how much charcoal is delivered in (1) day? (The answer) says: in one day 4 dou 2/[17] sheng of charcoal (is delivered). The method says: taking the 7 dou multiplied by 10 (days) gives 7 dan; and (it takes) 7 days as well to transport it to the post; i.e. take the 10 days and the 7 days and combine them as the divisor; (take 7 dan as the dividend); dividing gives the answer in dou.
“The meaning of this sentence [problem] is not clear…”
[Problem 47] 盧唐 Lu Tang: Bamboo Ladles/Utensils
[WW 2000, p. 82; PH 2001, p. 94; ZJS 2001, p. 265]

The norm says: in 1 day 60 stalks of bamboo are cut down; in 1 day (it is possible to make) 15 $lutang$, one stalk of bamboo equals 3 $lutang$. If 1 person is told to cut down bamboo himself to make $lutang$, how many can be made in 1 day? (The answer) says: 13 and 3/4 $lutang$. The method says: take 60 as the divisor, and take 55 times 15 as the dividend.

$$(55 \times 15) / 60 = 13 \ 3/4 \ lutang$$
郭世榮
Guo Shirong

《算數書》勘误《Suan shu shu》
kanwu (Corrections for the Suan shu shu),

内蒙古师范大学学报(自然科学汉文版)

\[
\frac{(60 \times 15)}{65} = 13 \frac{11}{13} \text{lutang/day}
\]

\[
\frac{(55 \times 15)}{60} = 13 \frac{3}{4} \text{lutang/day}
\]
算數書校勘 *Suan shu shu jiaokan* (Collation of the *Suan shu shu*), 中国科技史料 *Zhongguo kexu shiliao* (China Historical Materials of Science and Technology), 22(3)(2001), pp. 202-219.

\[
\frac{(60 \times 15)}{65} = 13 \frac{11}{13} \text{ lutang/day}
\]
張家山漢簡《算數書》注釋
Zhangjiashan hanjian Suan shu zhushi (Commentaries on the Book on Calculating with Numbers on Bamboo Strips, unearthed from Zhangjiashan), Beijing: Kexue chubanshe, 2001.

\[
(60 \times 15) / (60 + 15) = 12 \text{ lutang/day}
\]
To cut one stalk of bamboo takes $\frac{1}{60}$ of a day.
To make one lutang takes $\frac{1}{15}$ of a day.
One stalk of bamboo suffices for 3 lutang, which takes:
$\frac{1}{60} + 3(\frac{1}{15}) = \frac{13}{60}$ of a day.
4 stalks of bamboo will result in 12 lutang in $\frac{52}{60}$ of a day.
This leaves $\frac{8}{60}$ of the day. To cut one more bamboo $\frac{7}{60}$ of the day. It takes $\frac{4}{60}$ of leaves a day to make one lutang, resulting in 13 complete lutang, with $\frac{3}{60}$ of the day left.
Since it takes $\frac{4}{60}$ of a day to make one lutang, this leaves only enough time to complete $\frac{3}{4}$ of a lutang, for a total of $13\ 3/4$ lutang.
To cut one stalk of bamboo takes $\frac{1}{60}$ of a day. 
To make one lutang takes $\frac{1}{15}$ of a day.
One stalk of bamboo suffices for 3 lutang, which takes:
$\frac{1}{60} + 3(\frac{1}{15}) = \frac{13}{60}$ of a day.
4 stalks of bamboo will result in 12 lutang in $\frac{52}{60}$ of a day. 
This leaves $\frac{8}{60}$ of the day. To cut one more bamboo $\frac{7}{60}$ of the day. It takes $\frac{4}{60}$ of leaves a day to make one lutang, 
resulting in 13 complete lutang, with $\frac{3}{60}$ of the day left. 
Since it takes $\frac{4}{60}$ of a day to make one lutang, this leaves only enough time to complete $\frac{3}{4}$ of a lutang, for a total of 
$13 \frac{3}{4}$ lutang.

Thus, the answer as given is correct.
[Problem 47] 盧唐  *Lu Tang*: Bamboo Ladles/Utensils
[WW 2000, p. 82; PH 2001, p. 94; ZJS 2001, p. 265]

Thus the answer, 13 3/4 *lutang* is correct, but now, what about the method?

\[(55 \times 15)/60 = 13 \ 3/4 \ lutang\]
The norm: one person in one day makes 30 arrows or feathers 20 arrows. If one now wishes to have one person both make arrows and feather them, in 1 day how many can be made? The answer: 12. The method: combine the arrows and the feathering as divisor; taking the arrows and the feathering, mutually multiply them together as the dividend.
In one day 1 person can make 30 arrows, or feather 20 arrows. If one person does both, how many arrows can be made in one day?

\[ \frac{1}{30} + \frac{1}{20} = \frac{20+30}{20 \times 30} = \frac{50}{600} = \frac{1}{12} \text{ of a day,} \]
so that 12 arrows can be made and feathered in one day.

This is exactly what the method says: to total number of feathered arrows will be:

\[ \frac{20 \times 30}{20+30} = \frac{600}{50} = 12 \text{ feathered arrows/day.} \]
[Problem 47] 盧唐 *Lu Tang*: Bamboo Ladles/Utensils
[WW 2000, p. 82; PH 2001, p. 94; ZJS 2001, p. 265]

Why doesn’t $(60 \times 15)/(60+15) = 12$ *lutang* work?

Correct method:

$$\frac{(55 \times 15)}{60} = 13 \frac{3}{4} *lutang$$
In 1 day 1 person can collect 7 dou of charcoal
The next day it is possible to deliver 1 dan (10 dou) of charcoal
Thus it takes 1/7 of a day to collect 1 dou of charcoal and 1/10 of a day to deliver 1 dou: 1/7 + 1/10 = (10+7)/70, i.e. it will take 17/70 of the day to gather and deliver 1 dou of charcoal; therefore, in one day 70/17 dou can be gathered and delivered, i.e. = 4 2/17 dou/day.

Alternatively:
If 7 dou can be collected in one day, in 10 days, 70 dou or 7 dan can be collected; to deliver 7 dan will take 7 days, so altogether, 70 dou collected and delivered in 17 days amounts to:

70 dou/17 days = 4 2/17 dou/day.
九章算術 Jiu zhang suan shu
Nine Chapters on the Art of Mathematics
九章算術  *Nine Chapters on the Art of Mathematics*

- Alexander Wylie (1852): *Arithmetical Rules of the Nine Sections*
- Mikami Yoshio (1913) *Arithmetic in Nine Sections.*
- David Eugene Smith (1923): *Arithmetic in Nine Sections*
- George Sarton (1927): *Arithmetic in Nine Sections*
- Joseph Needham (1959): *Nine Chapters on the Mathematical Art*
- Jean-Claude Matzloff (1987): *Computational Prescriptions in Nine Chapters*
- Lam Lay-Yong (1994): *Nine Chapters on the Mathematical Art*
- Shen (Crossley and Lun) (1999): *Nine Chapters on the Mathematical Art*
- Alexei Volkov (2007): *Computational Procedures of Nine Categories*
The categories under which the matters [treated herein fall] extend each other [when compared], so that each benefits [from the comparison]. So even though the branches are separate they come from the same root, and one may know that they each show a separate tip [of the same tree].

[Cullen 2002: 788]
Therefore one studies similar methods in comparison with each other, and one examines similar affairs in comparison with each other. This is what makes the difference between stupid and intelligent scholars, between the worthy and the unworthy. Therefore, it is the ability to distinguish categories in order to unite categories (neng lei yi he lei 能類以合類) which is the substance of how the worthy one’s scholarly patrimony is pure, and of how he applies himself to the practice of understanding.”

[Cullen 2002: 789]
九章算術  Liu Hui versus Euclid’s *Elements*

Whereas Euclid was concerned to show how a great number of true propositions could be deduced from a small number of axioms, the anonymous author of the *Jiu zhang* followed a different but no less rational route in the reverse direction. He started from the almost infinite variety of possible problems and aimed to show that those known to him could all be reduced to nine basic categories solvable by nine basic methods.

[Cullen 2002: 789]
US TEAM MAKES HISTORY AT INTERNATIONAL MATHEMATICAL OLYMPIAD

Competing against teams representing 69 countries, a team of six American high school students placed first in the 35th International Mathematical Olympiad with six perfect scores. This is the first time in the 35-year history of the Olympiad that any team has achieved a perfect score. Each US team member scored the maximum number of points (42) on the nine-hour competition, which took place July 8-20 in Hong Kong, and each received a gold medal. The members of the team are: Jeremy Bem, Ithaca, NY; Aleksandr Khazanov, New York, NY; Jacob Lurie, Silver Spring, MD; Noam Shazeer, Swampscott, MA; Stephen Wang, Aurora, IL; and Jonathan Weinstein, Lexington, MA. The teams placing second through fifth are from China, Russia, Bulgaria, and Hungary, in that order. The US team was chosen on the basis of performance in the 23rd annual USA Mathematical Olympiad held earlier this year, and then participated in a month-long summer program at the US Naval Academy [MAA AMC 1994].
It is especially appropriate to honor these outstanding students as the Senate prepares to consider the Improving America's Schools Act. Our bill places particular emphasis on math and science education, and provides strong Federal support to strengthen math and science instruction in schools throughout America. Not every student can equal the brilliant and inspiring accomplishments of our Olympiad team, but all students deserve an education that develops their full potential.
Jeremy Bem, Member of the USA IMO team, Summer 1994. Photo by Kimberly Butler//Time Life Pictures/Getty Images
Eric Lander
(Stuyvesant High School, New York City)

Math team was great. About thirty kids met each morning for an hour before school in a fifth-floor room of Stuyvesant High School, and the captain of the team was responsible for running the session. This was before you had databases full of math problems, so the captain of the math team, upon his ascension to office, came into possession of what we called “the shopping bag.” It contained mimeographed sheets of problems and strips of problems and records of the city math contests for a long time. So the captain of the team would pull problems out of the bag and be responsible for leading the group.

[Olson 2005: 3–4]
Solving these problems requires a sophisticated grasp of mathematical ideas, so that familiar concepts can be extended in new directions. So the mathematical procedures everyone learns in school aren’t enough. Becoming an excellent problem solver demands creativity, daring, and playfulness. A math competition is more like a game than a test—a game played with the mind.

[Olson 2005: 3]
Practice, practice, practice. The only way to learn math is by doing
Read the masters.
There’s more than one road. Different solutions can be equally valid; differences in perspective can be significant and valuable
It’s not over when it’s over. Keep thinking about problems
Learn from your peers. They’re smarter than you might have expected
Learn from the past. Try to relate new problems to old ones
Patience. No one said this was easy!
The History of Mathematics Teaching
The Tension Between Practice and Theory

Victor Katz

Joseph W. Dauben
Department of History
Herbert H. Lehman College
and
Ph.D. Program in History
The Graduate Center
City University of New York