Mathematics and the drift towards purity

or

Why mathematics education is romantic

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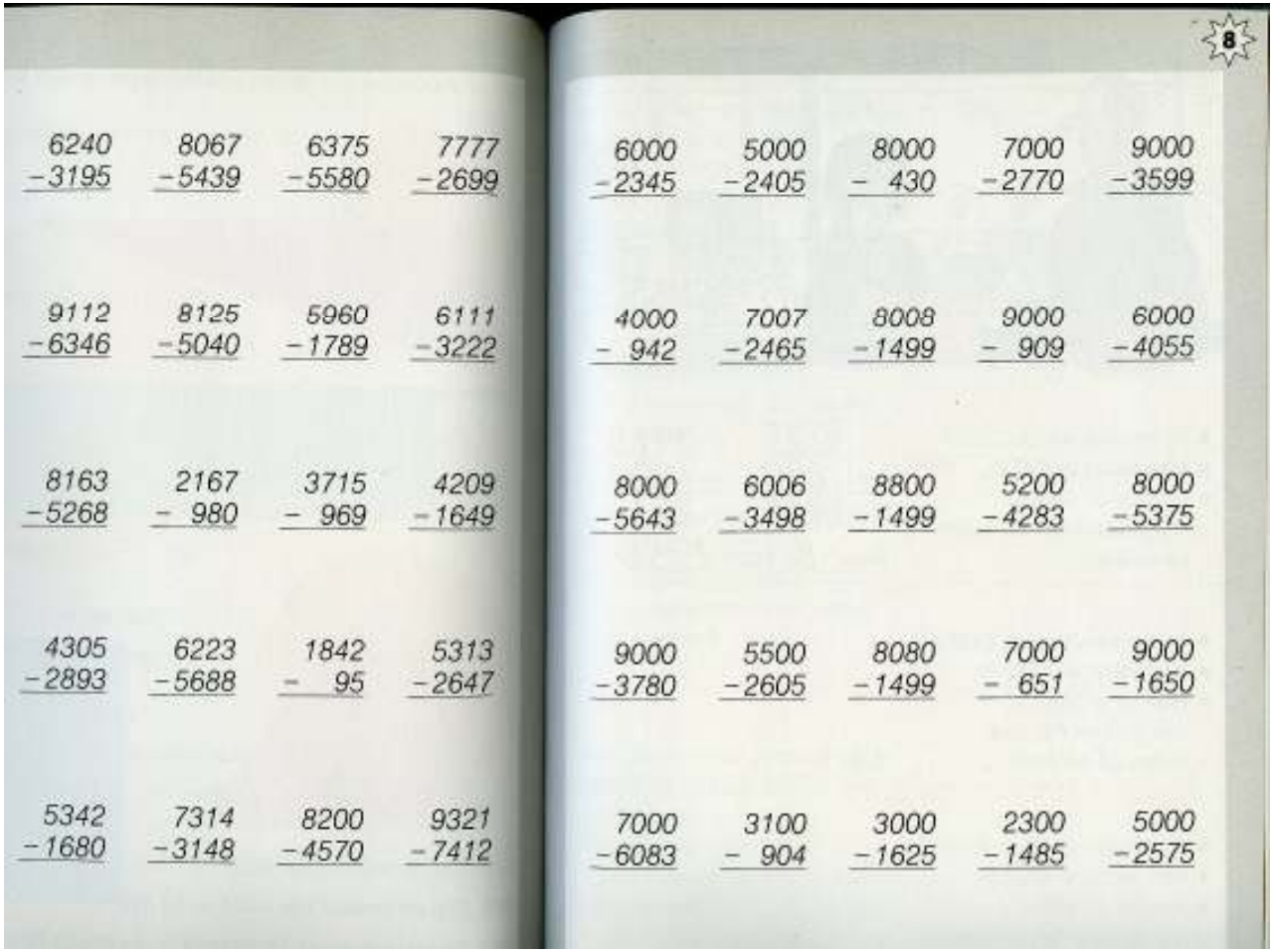
1. A problem
2. A historical explanation
3. Consequences of this history for how we think today about the relationship between learning and technology

- To think mathematically affords a powerful means to understand and control one's social and physical reality.
- Yet despite some 12 or so years of compulsory mathematical education, most children in the developed world leave school with only a limited access to mathematical ideas.
  - Noss & Hoyles, 1996

- As a teacher, you will have the awesome responsibility of helping *all* of your students construct the disposition and knowledge needed to live successfully in a complex and rapidly changing world. To meet the challenges of the twenty-first century, students will especially need *mathematical power* […]
- Unfortunately, the mathematics curriculum, the instructional practices and assessment methods that you might be accustomed to […] are generally not effective in promoting children's mathematical power.
  - Baroody och Coslick, 1998

Is this surprising?



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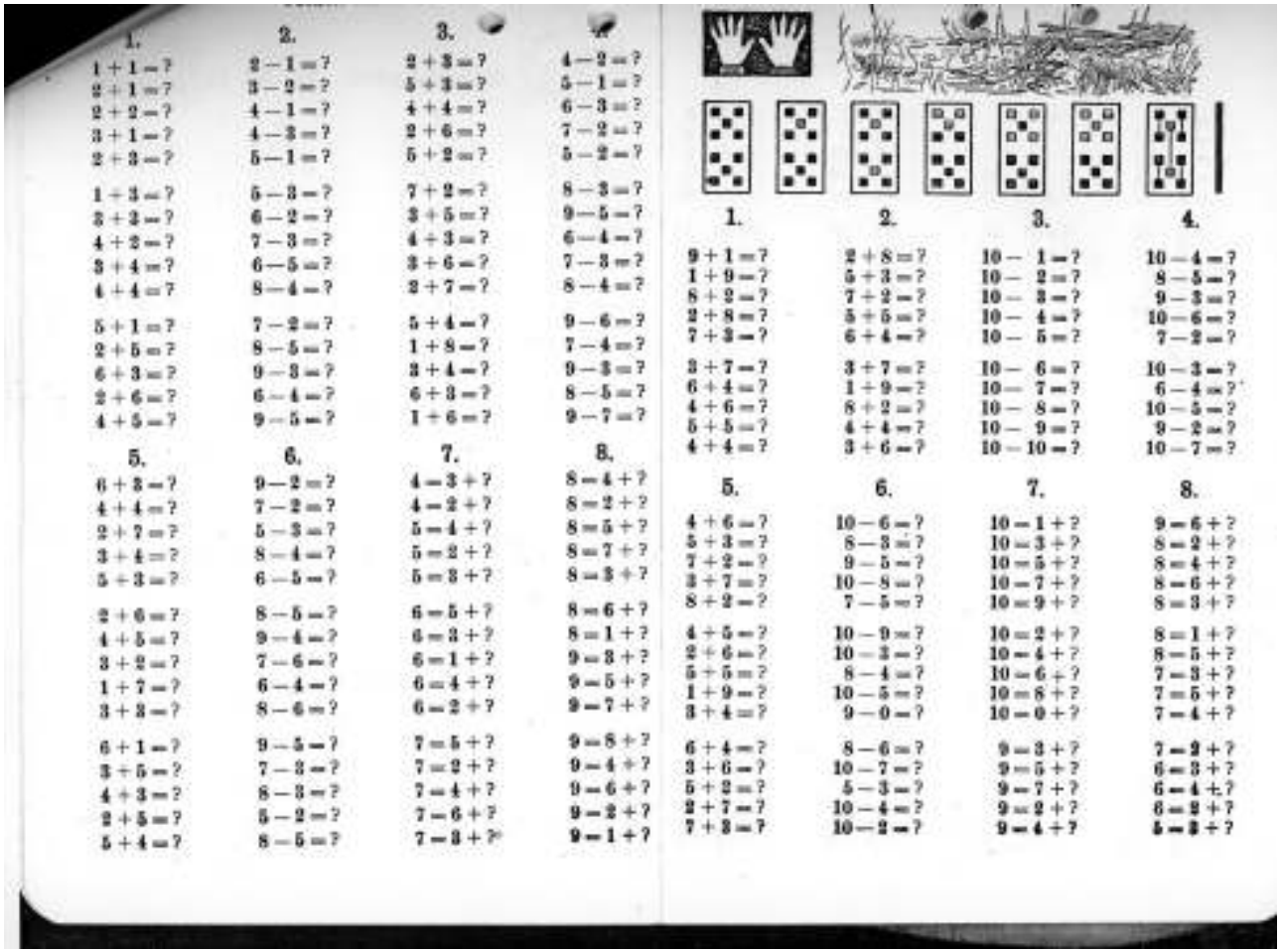
−3195	−5439	−5580	−2699	−2345	−2405	− 430	−2770	−3599
9112	8125	5960	6111	4000	7007	8008	9000	6000
−6346	−5040	−1789	−3222	− 942	−2465	−1499	− 909	−4055
8163	2167	3715	4209	8000	6006	8800	5200	8000
−5268	− 980	− 969	−1649	−5643	−3498	−1499	−4283	−5375
4305	6223	1842	5313	9000	5500	8080	7000	9000
−2893	−5688	− 95	−2647	−3780	−2605	−1499	− 651	−1650
5342	7314	8200	9321	7000	3100	3000	2300	5000
−1680	−3148	−4570	−7412	−6083	− 904	−1625	−1485	−2575

- Arithmetic is universally taught in schools, but almost invariably as the art of mechanical computation only.
- The true significance and the symbolism of the processes employed are concealed from pupil and teacher alike.
- The subtlety, delicacy, and accuracy of mathematical processes have the highest educational value, both direct and indirect.
- To treat them as mechanical routine, not susceptible of explanation or illumination from a higher point of view, is to destroy in large measure the value of mathematics as an educational instrument, and to aid in arresting the mental development of the pupil.
  - David Eugene Smith, *The Teaching of Elementary Mathematics,* 1900

1920s



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4 + 3 = ?	8 − 3 = ?	7 − 4 + ?	9 − 6 + ?
2 + 5 = ?	5 − 2 = ?	7 − 6 + ?	9 − 2 + ?
5 + 4 = ?	8 − 5 = ?	7 − 3 + ?	9 − 1 + ?

1.	2.	3.	4.
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8 + 2 = ?	7 + 2 = ?	10 − 3 = ?	9 − 3 = ?
2 + 8 = ?	5 + 5 = ?	10 − 4 = ?	10 − 6 = ?
7 + 3 = ?	6 + 4 = ?	10 − 5 = ?	7 − 2 = ?
3 + 7 = ?	3 + 7 = ?	10 − 6 = ?	10 − 3 = ?
6 + 4 = ?	1 + 9 = ?	10 − 7 = ?	6 − 4 = ?
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5.	**6.**	**7.**	**8.**
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5 + 3 = ?	8 − 3 = ?	10 − 3 + ?	8 − 2 + ?
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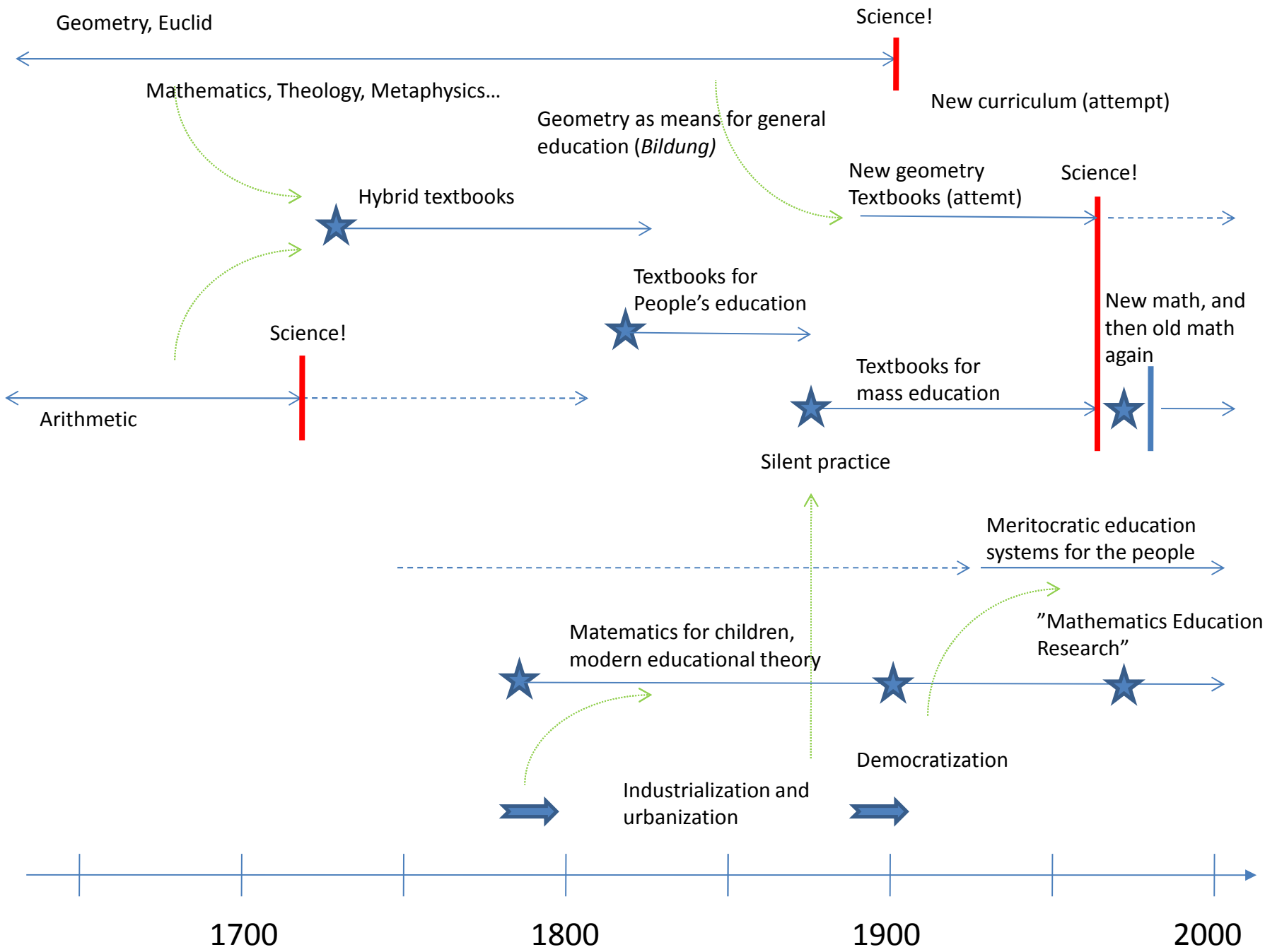
3 + 7 = ?	10 − 8 = ?	10 − 7 + ?	8 − 6 + ?
8 + 2 = ?	7 − 5 = ?	10 − 9 + ?	8 − 3 + ?
4 + 5 = ?	10 − 9 = ?	10 − 2 + ?	8 − 1 + ?
2 + 6 = ?	10 − 3 = ?	10 − 4 + ?	8 − 5 + ?
5 + 5 = ?	8 − 4 = ?	10 − 6 + ?	7 − 3 + ?
1 + 9 = ?	10 − 5 = ?	10 − 8 + ?	7 − 5 + ?
3 + 4 = ?	9 − 0 = ?	10 − 0 + ?	7 − 4 + ?
6 + 4 = ?	8 − 6 = ?	9 − 3 + ?	7 − 2 + ?
3 + 6 = ?	10 − 7 = ?	9 − 5 + ?	6 − 3 + ?
5 + 2 = ?	5 − 3 = ?	9 − 7 + ?	6 − 4 + ?
2 + 7 = ?	10 − 4 = ?	9 − 2 + ?	6 − 2 + ?
7 + 3 = ?	10 − 2 = ?	9 − 4 + ?	5 − 3 + ?

The ”Standard Critique”

- Higher goals not directly related to mathematics are to be reached through realistic, relevant, interesting problem solving leading to conceptual understanding.
- Claims that mathematics education how it is actually taught does not lead to these goals. Complaints about mechanism, memorization, rote learning, rule following.
- Calls for reform. Belief that *now* is the time when everything will finally change.
- (But is does not.)

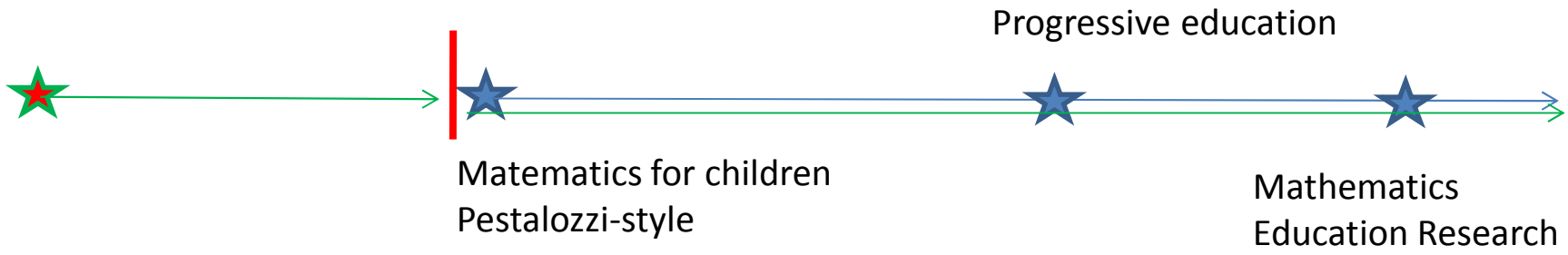
Where does this come from?

Part 2 - history


Geometry, Euclid
Science!
Mathematics, Theology, Metaphysics…
New curriculum (attempt)
Geometry as means for general education (*Bildung)*
New geometry Textbooks (attemt)
Science!
Hybrid textbooks
Textbooks for People's education
New math, and then old math again
Science!
Textbooks for mass education
Arithmetic
Silent practice
Meritocratic education systems for the people
Matematics for children, modern educational theory
"Mathematics Education Research"
Democratization
Industrialization and urbanization
1700
1800
1900
2000

History is complex

It is necessary to simplify



Enlightenment
Abstraction
Proof
Memory
Explanation
Facts
Efficiency
Application
Technology

Romantic reaction (Christian)
Intuition
Sense experience
Physical activity
Realism
Contextualization
Creativity
No explanation
No technology

- Calculating mechanically is when you follow rules without insight and understanding. Everyone, who does this, is therefore a machine, like a clock, a steam engine or any other machine. But man should not be a machine; she belongs to the organic creatures. To train a child by means of sense-less calculation, by means of play with dead digits, is to dehumanize her, to schackel or kill her spirit. It is an intellectual homicide.
  - Adolph Diesterweg, 1830

Split mathematics

Bad for learning	Good for learning
Formalism	Realism
Following rules	Creativity
Memorization	Intuition
Learning facts	Sense experience
Training skills	Physical activity
Being quick	Being slow
Being told, being explained to	No explanation (find out by yourself)
Reading and Listening	No use of tools (struggle is learning)
Use of tools	Understanding

While we perhaps do not ”personally” simplify in this way…
the dichotomy is built into the structure of our education systems.
It is reflected in educational theory.
It is expressed in the ”standard critique” and sets the direction for reform.

”The drift towards purity”

- Yes, the reaction against enlightenment was about purity! But it is important to understand *what kind* of purity.
- It was a purity in *separation* from modern scientific and technological society; from efficiency and instrumentality.
- The vision was that work with mathematics would put the child into a harmonic, ”pure”, relationship with her environment. For this, ”realism” was crucial – both for understanding the presence of mathematics in the world, and for understanding the usefulness of mathematical knowledge.
- The central idea was that mathematical knowledge **must** come from the inside, it must be discovered, grown, built. This made the activity of *problem solving* central for learning. Tools and explanations was considered to distract from this process.

What does this mean today?
Part 3 - technology

A history of reluctance

New technology	Prefered in mathematics education
Pen and paper	Mental arithmetic
Decimals	General fractions
Algebra and equations	Intuitive methods
Pocket calculators	Calculation by hand
Computers	Calculation by hand

- From the perspective of modern mathematics education, tools and technology seems to constitute an obstacle to learning, because it removes the ”problem” from many of the problems solved in school.
- But the standard critique indicates that the work-without-tools that pupils perform in school look very different depending on perspective:
  - In the past and present it usually looks like calculation
  - Only in the future - as envisioned! – does it looks like interesting, fun, relevant problem solving leading to useful knowledge.

Conclusions?

1. Be careful with the higher goals. (democracy, thinking)
2. Think about *how* what students actually *do* in school, could leads to these goals. Expect no magic from the subject matter of mathematics.
3. Open up for other ways of approaching mathematics than through tool-free, ”pure”, problem solving:
    1. By for instance reading about its history and sociology - its relationships to theology and technology alike - and discussing it. This would make mathematics education *educating* in a way that problem solving can never be.
    2. By allowing the students to use proper tools when working with problem solving.
4. Recognize that we have perhaps not only inherited a *problem* from history, but perhaps also, less visibly, its ”*solution*”. We need to rethink both.