Mathematics Standards Of PISA Countries

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Standards Reviewed

- Australia (New South Wales, Victoria)
- Canada
  - ➤ Alberta
  - ➤ Ontario
- China
  - ➤ Shanghai
- Finland
- Korea
- US

Most Common Trends

1. Equality of Opportunity---Mathematics for All

2. Integration of Mathematical Processes with Content Coverage

3. Incorporation of Local Issues in Curricular Materials

4. Utilization of Multiple Mathematical Contexts

5. Use of Digitally Based Resources

1 - Equality of Opportunity

- **Strive for equality for ALL students**
  - ➢ Equality in content coverage across regions of the country
  - ➢ Accommodation of different learning styles
  - ➢ Accommodation of diverse abilities and backgrounds
  - ➢ Content coverage that defines Mathematics for All
  - ➢ Curricula that addresses different student needs

2 - Integration of Mathematical Processes with Content Coverage

- Problem Solving
- Communication Around Mathematical Ideas
- Mathematical Reasoning
- Mathematical Modeling

2a - Problem Solving

- Read and understand
  - What is known?
  - Draw a picture; make a list, table, graph, or chart
- Plan approach
  - Look for a pattern
  - Choose from algorithms previously used
  - OR develop a strategy
- Solve
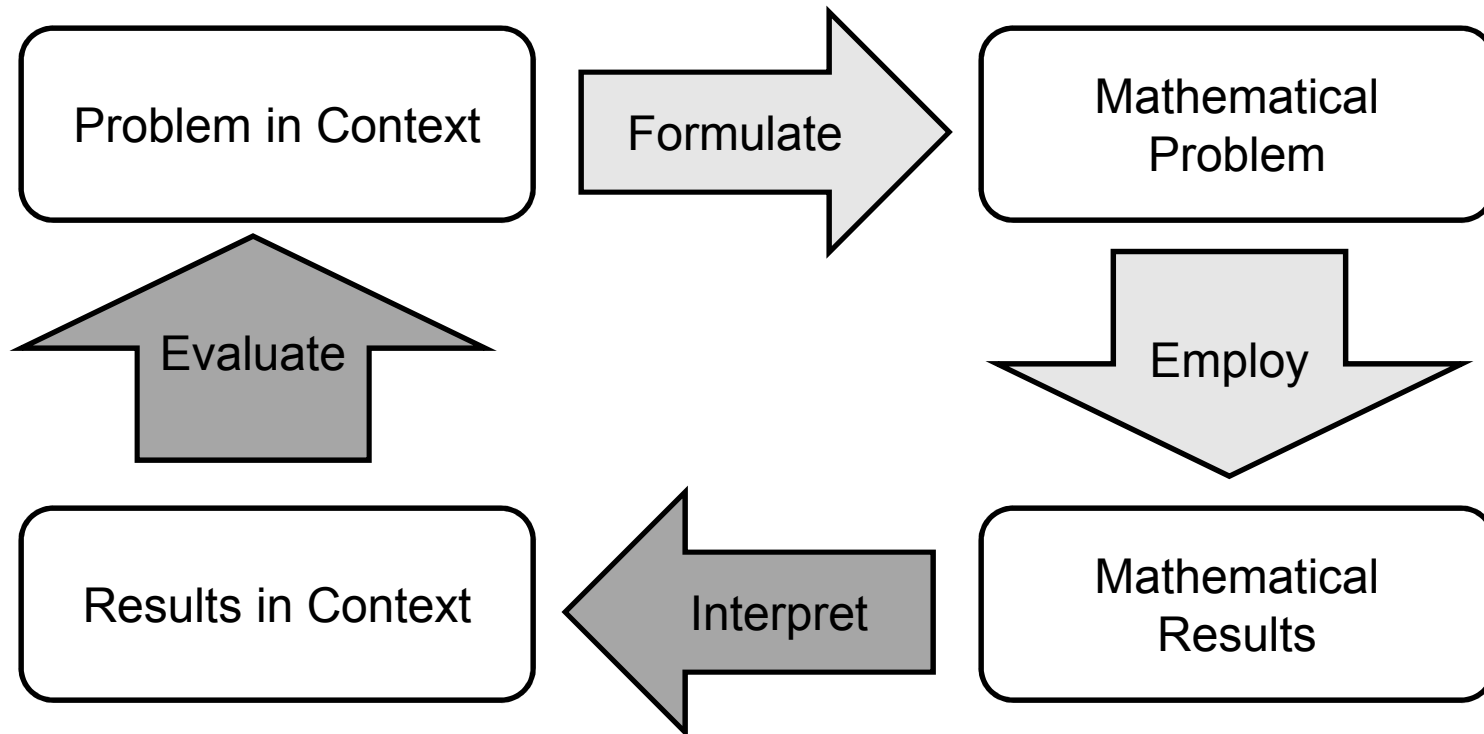- Verify reasoning; verify calculations

2b - Communication Around Mathematical Ideas

- Build mathematics vocabulary
- Students describe approaches to problem solving
- Students demonstrate ability to communicate mathematical ideas orally, visually, and in writing
- Students use precise mathematical language

2c - Mathematical Reasoning

- Build abilities to reason, think logically, complete proofs, and justify conclusions

- Promote creative thinking, allowing students to explore and discover mathematical concepts

- Ensure students can answer mathematical questions: Why? How? What if? How do you know that?

2d - Mathematical Modeling

Problem in Context → Formulate → Mathematical Problem → Employ → Mathematical Results → Interpret → Results in Context → Evaluate → Problem in Context

OECD (2013), *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy,* Figure 1.1, p. 26

School Education System in China

18-	University, college, high vocational schools, employment
	Gaokao
15-17	Senior high school, secondary vocational school
	Zhongkao

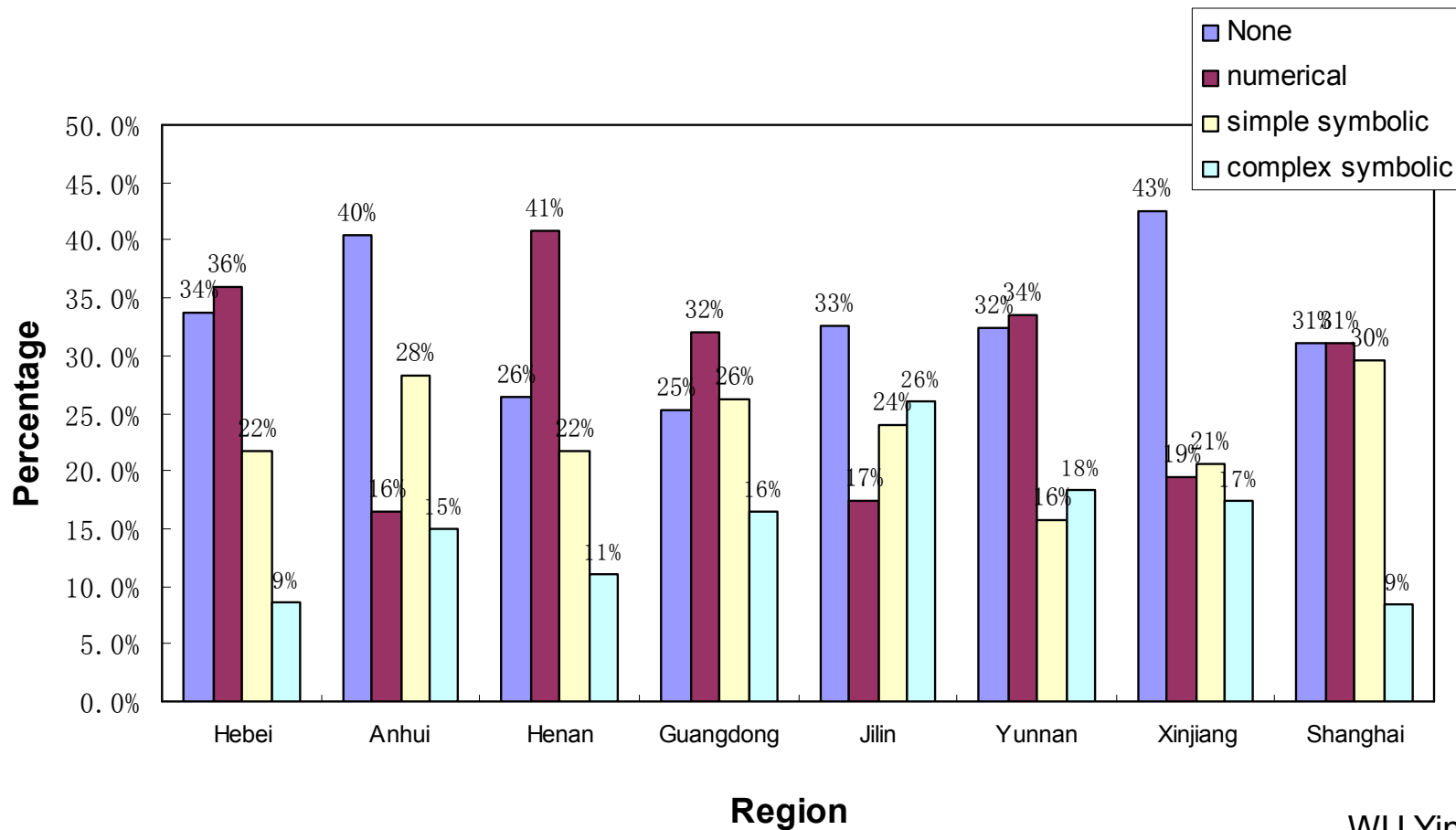| 6-15 | 13-15 (12-15) | Junior high school |
| | 6-12 (6-11) | Elementary school |
| | Nine-year compulsory education | |

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Percentages of Items at Different Levels of Cognitive Demand Across Regions
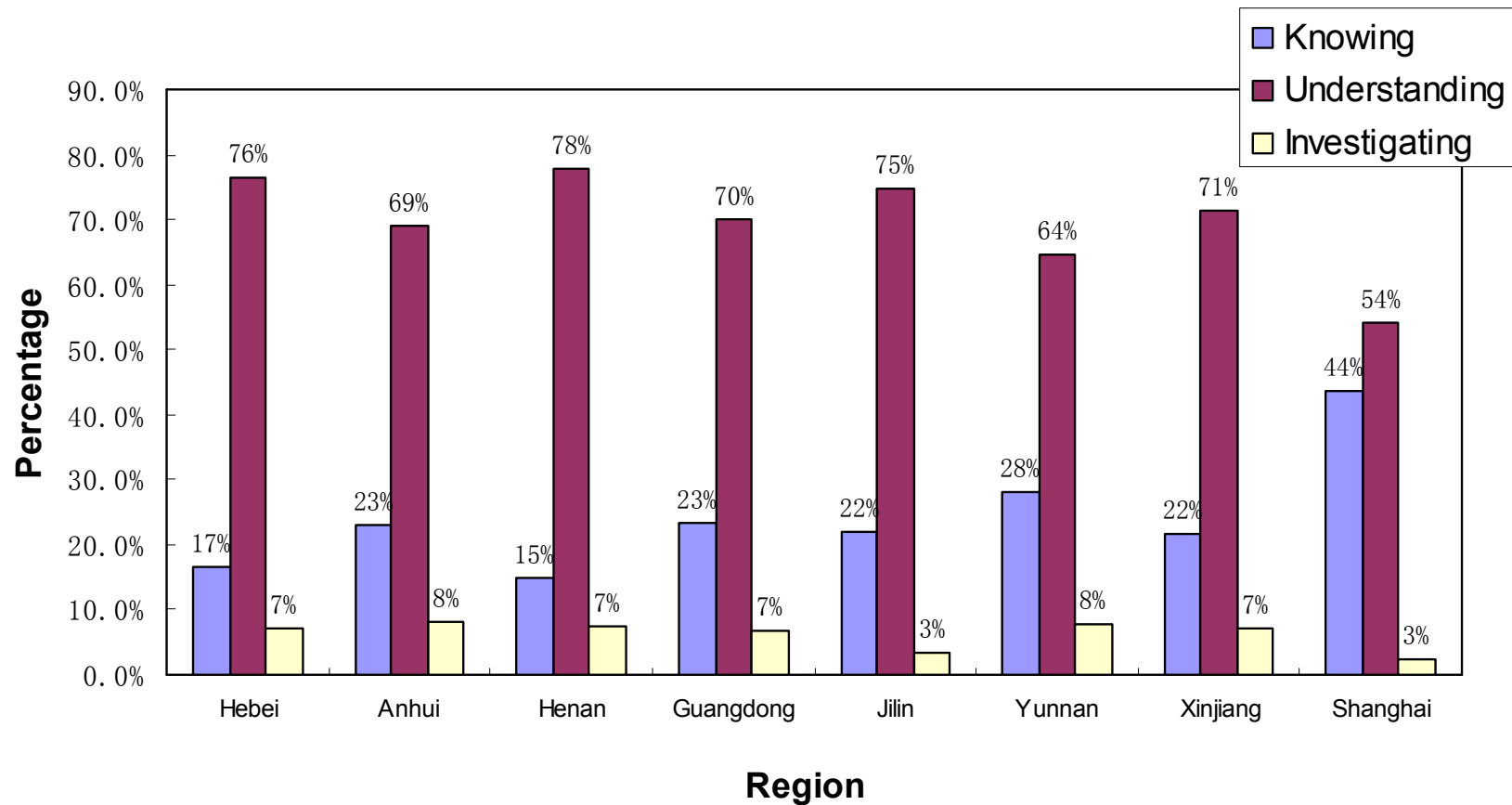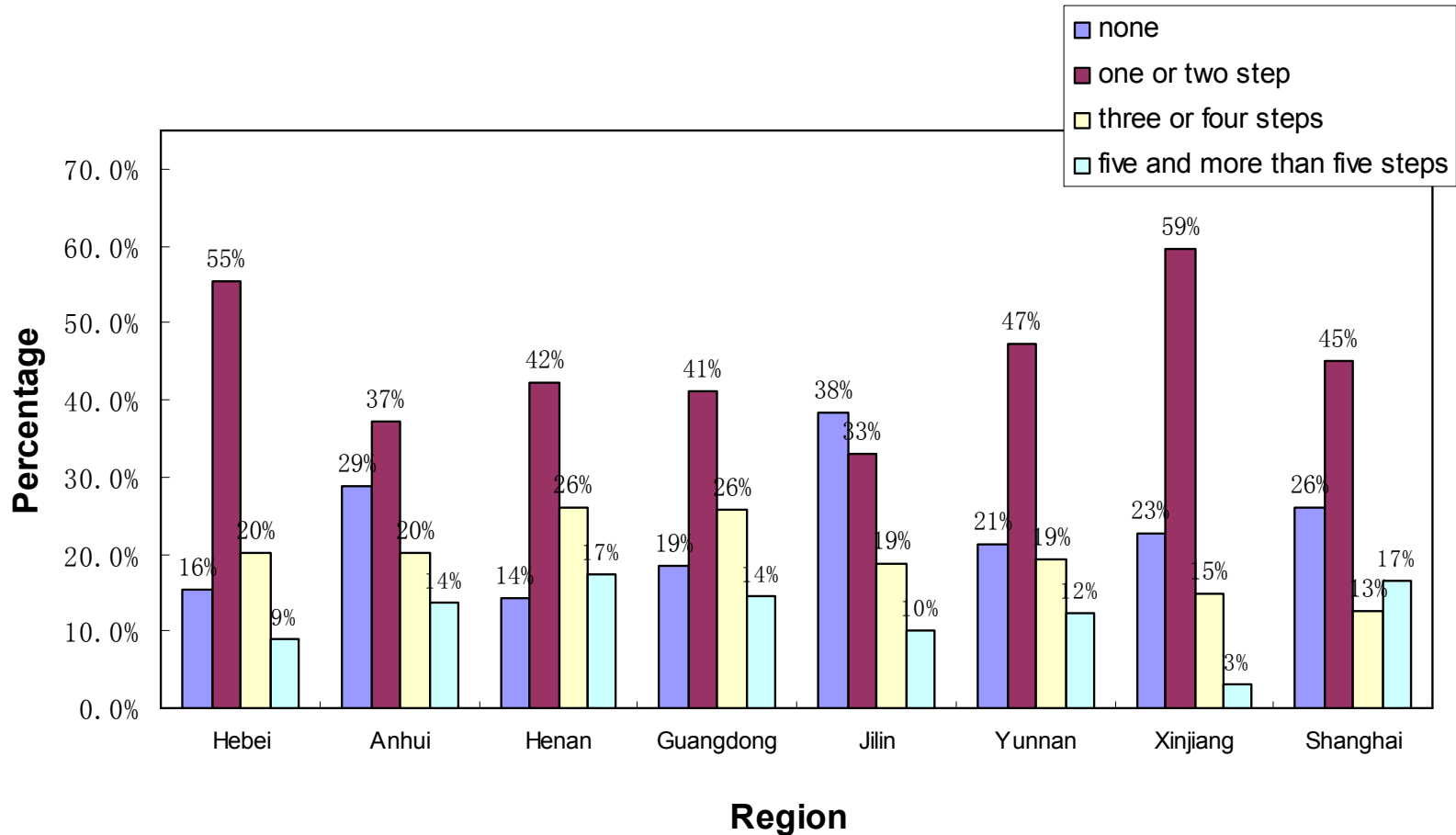

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Percentages of Items at Different Levels of Computation Across Regions

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Percentages of Items at Different Levels of Reasoning Across Regions

Legend:
- none
- one or two step
- three or four steps
- five and more than five steps

Hebei: 16%, 55%, 20%, 9%
Anhui: 29%, 37%, 20%, 14%
Henan: 14%, 42%, 26%, 17%
Guangdong: 19%, 41%, 26%, 14%
Jilin: 38%, 33%, 19%, 10%
Yunnan: 21%, 47%, 19%, 12%
Xinjiang: 23%, 59%, 15%, 3%
Shanghai: 26%, 45%, 13%, 17%

Y-axis: Percentage (0.0% – 70.0%)
X-axis: Region

Mathematical Processes--US

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Make Sense of Problems and Persevere in Solving Them

- Understand the meaning of the problem and look for entry points to its solution
- Analyze information (givens, constrains, relationships, goals)
- Make conjectures and plan a solution pathway
- Monitor and evaluate the progress and change course as necessary
- Check answers to problems and ask, "Does this make sense?"
- Try special cases or simpler forms to gain insight

Reason Abstractly and Quantitatively

- Make sense of quantities and relationships in problem situations

- Represent abstract situations symbolically and understand the meaning of quantities

- Create a coherent representation of the problem at hand

- Consider the units involved

- Flexibly use properties of operations

Construct Viable Arguments and Critique the Reasoning of Others

- Use definitions and previously established causes/effects (results) in constructing arguments
- Make conjectures and use counterexamples to build a logical progression of statements to explore and support their ideas
- Communicate and defend mathematical reasoning using objects, drawings, diagrams, actions
- Listen to or read the arguments of others
- Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments
- Compare the effectiveness of two plausible arguments

Model with Mathematics

- Apply prior knowledge to solve real world problems

- Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas

- Make assumptions and approximations to make a problem simpler

- Check to see if an answer makes sense within the context of a situation and change a model when necessary

- Use symbols to represent real world problems

Use Appropriate Tools Strategically

- Make sound decisions about the use of specific tools Examples might include:
  - ➢ Calculator
  - ➢ Concrete models
  - ➢ Digital Technology
  - ➢ Pencil/paper
  - ➢ Ruler, compass, protractor

- Use technological tools to visualize the results of assumptions, explore consequences and compare predications with data

- Know the limits of each tool for providing accurate solutions for the problem

- Detect tool-generated errors by estimating reasonable solutions without the tool

Attend to Precision

- Communicate precisely using clear definitions

- State the meaning of symbols, carefully specifying units of measure, and providing accurate labels

- Calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate to the problem context

- Provide carefully formulated explanations

- Label accurately when measuring and graphing

- Express numerical answers with a degree of precision appropriate to the problem context

Look For and Make Use of Structure

- Look for both numerical and visual patterns or structure, recognizing that quantities can be represented in different ways

- Recognize the significance in concepts and models and use the patterns or structure for solving related problems

- View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems

- Students' attention is consistently drawn to the structure of the mathematics as it occurs

Look For and Express Regularity in Repeated Reasoning

- Notice repeated calculations and look for general methods and shortcuts
- Continually evaluate the reasonableness of intermediate results (comparing estimates) while attending to details and make generalizations based on findings

3 - Incorporation of Local Issues in Curricular Materials

- Encourage positive attitudes about mathematics -- link mathematics to real world situations

- Incorporate local flavor to curricular planning; seek input from teachers, administrators, parents and business leaders

4 - Utilization of Multiple Mathematical Contexts

- Promote connections between mathematical concepts

- Promote connections between math and other disciplines

- Promote meaningful connections between math and the real world

- Develop a strong sense of citizenship through the use of mathematics

Percentages of Items at Different Levels of Context Across Regions

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Center for the Study of Curriculum
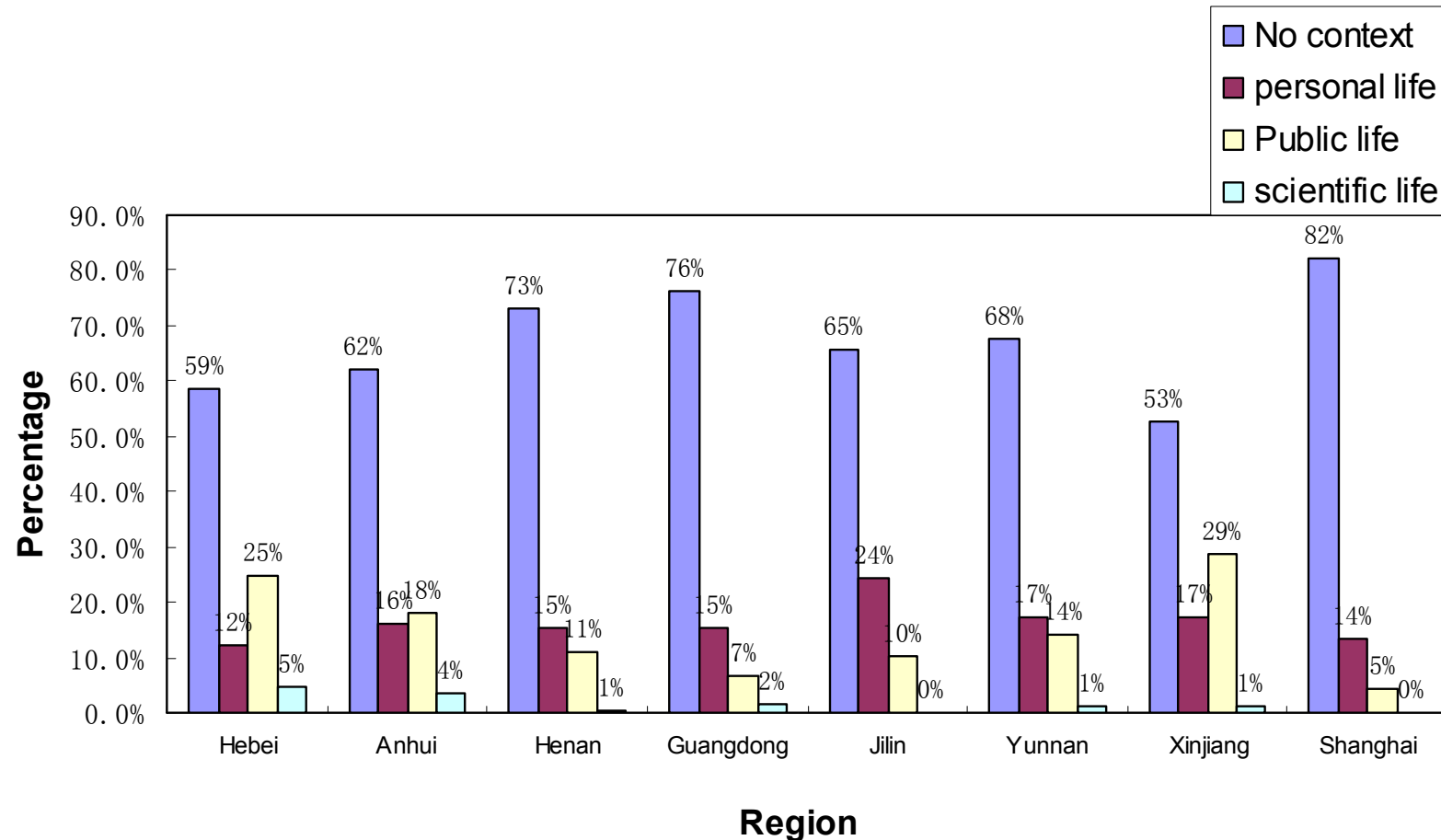
5 - Digitally Based Resources

- Keep up with technological advances in classroom instruction
  - Use electronic tools readily
  - Educational broadcasts (radio, television cable)
  - Supplement text material with available digital resources
    - Computer software
   - Web-based instruction / lessons

Curricular Trends: What Countries?

Emphasized in Curriculum	Australia	Alberta, CA	Ontario, CA	Finland	Korea	Shanghai
Equality	●	●	●	●	●	●
Integrate mathematical processes	●	●	●	●	●	●
• Problem solving	●	●	●	●	●	●
• Communication	●	●	●	●		●
• Mathematical Reasoning	●	●	●	●	●	●
Local Issues	●	●	●	●	●	●
Encourage Connections		●		●	●	
Utilize digitally based resources	●	●	●	●	●	●

Coverage of Key Content: K-8

Strand/Topic - Grade First Intended	Australia (NSW)	Alberta, CA	Ontario, CA	Finland	Korea	Shanghai
Adding Fractions, unlike denom	7	7	7	3-5	5	4
Unit Rates & Proportionality	7	8	7 / 8	6-8	5	6-7
Identifying Numeric Patterns	K / 1	2	1	3-5	2	1-2
Solving Linear Equations	7	7	7	6-8	7	6-7
Quadratic Equations	9	9	> 8	6-9	9	6-7
Coordinate System	7	6	5	3-5	7	7
Concept of Volume	K / 1	5	4	6-8	5	5
Finding Volume	7	6	5 / 7	6-8	5 / 6	5
Pythagorean Theorem	7	8	8	6-8	9	8
Symmetry	1 / 2	4	2	3-5	5	7
Congruence	7	4	3	3-5	5	7
Similarity	7	9	7	3-5	8	8-9
Measures of Central Tendency and Range	7	7	5	3-5	5	6
Probability	5 - 7	6 / 7	5	6-8	8	5

Alberta High School Tracks
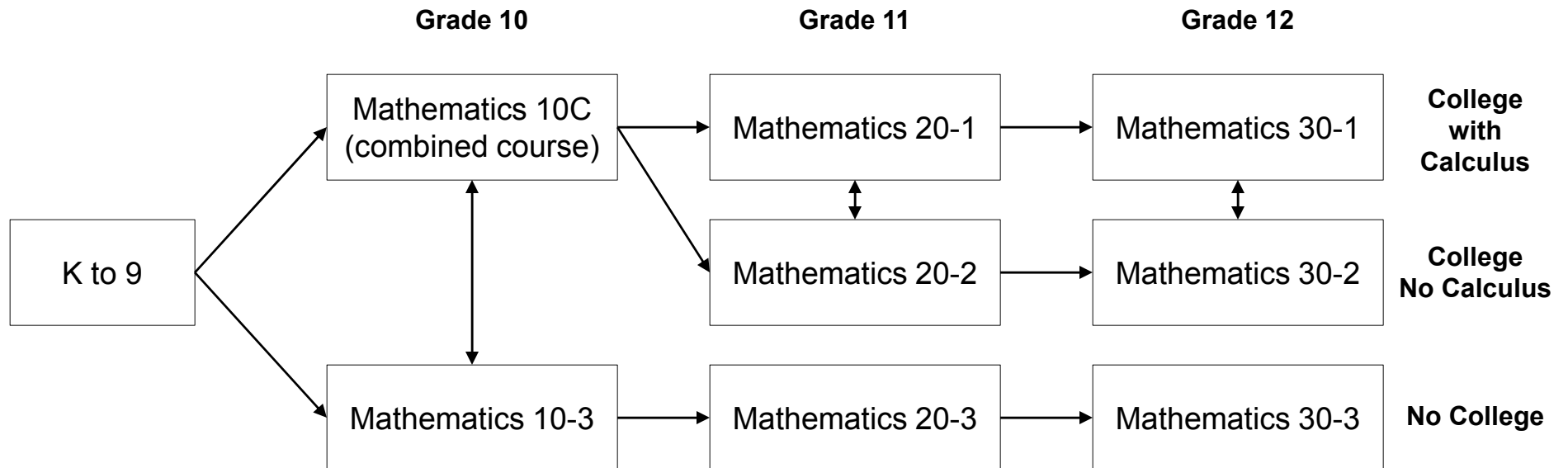
Coverage of Key Content: High School

Topic	Alberta, Canada			Finland		Korea	Shanghai
	College with Calculus	College NO Calculus	No College	College	No College		
Graph and analyze exponential and logarithmic functions	●	●		●	●	●	●
Solve systems of equations	●			●	●	●	●
Apply translations & stretches to functions (graphs & equations)	●					●	
Find the distance between two points (Pythagorean Theorem)			●	●	●	●	●
Understand inverses of relations	●			●		●	●
Graph and analyze quadratic functions	●	●		●	●	●	●
Graph and analyze trigonometric functions	●			●		●	●
Perform operations with composite functions	●			●		●	●
Understand transformations on a 2D shape or 3D			●			●	
Perform operations with vectors; vector calculus				●		●	●
Find limit of sequences and series						●	●
Find limit of a trigonometric function						●	
Find derivative of a polynomial function				●	●	●	
Integrate a polynomial function				●		●	
Apply set theory to solve problems		●					
Calculate permutations and combinations	●	●		●	●	●	●

Notable Differences in Grade Level Coverage K-8

- Coordinate System 3-7
- Concept of Volume K-6
- Symmetry 1-7
- Congruence 3-7
- Similarity 5-9

Potential Secondary Topics For All

- Pythagorean Theorem
- Exponential/Logarithmic Functions
- Quadratic Functions
- Permutations and Combinations
- No Others out of the 16