PISA'S VIEW ON MATHEMATICS

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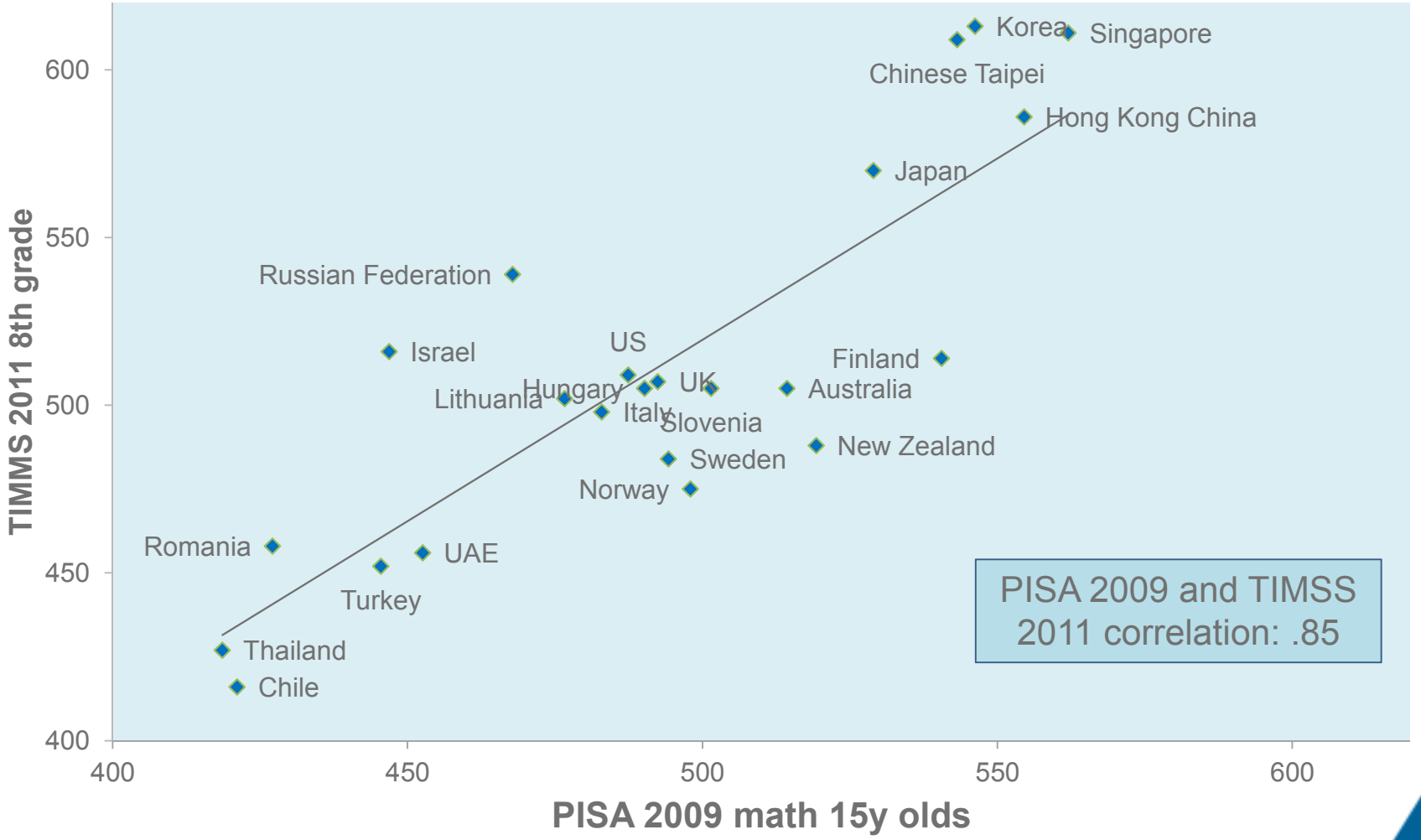
Comparing TIMSS 2011 and PISA 2009



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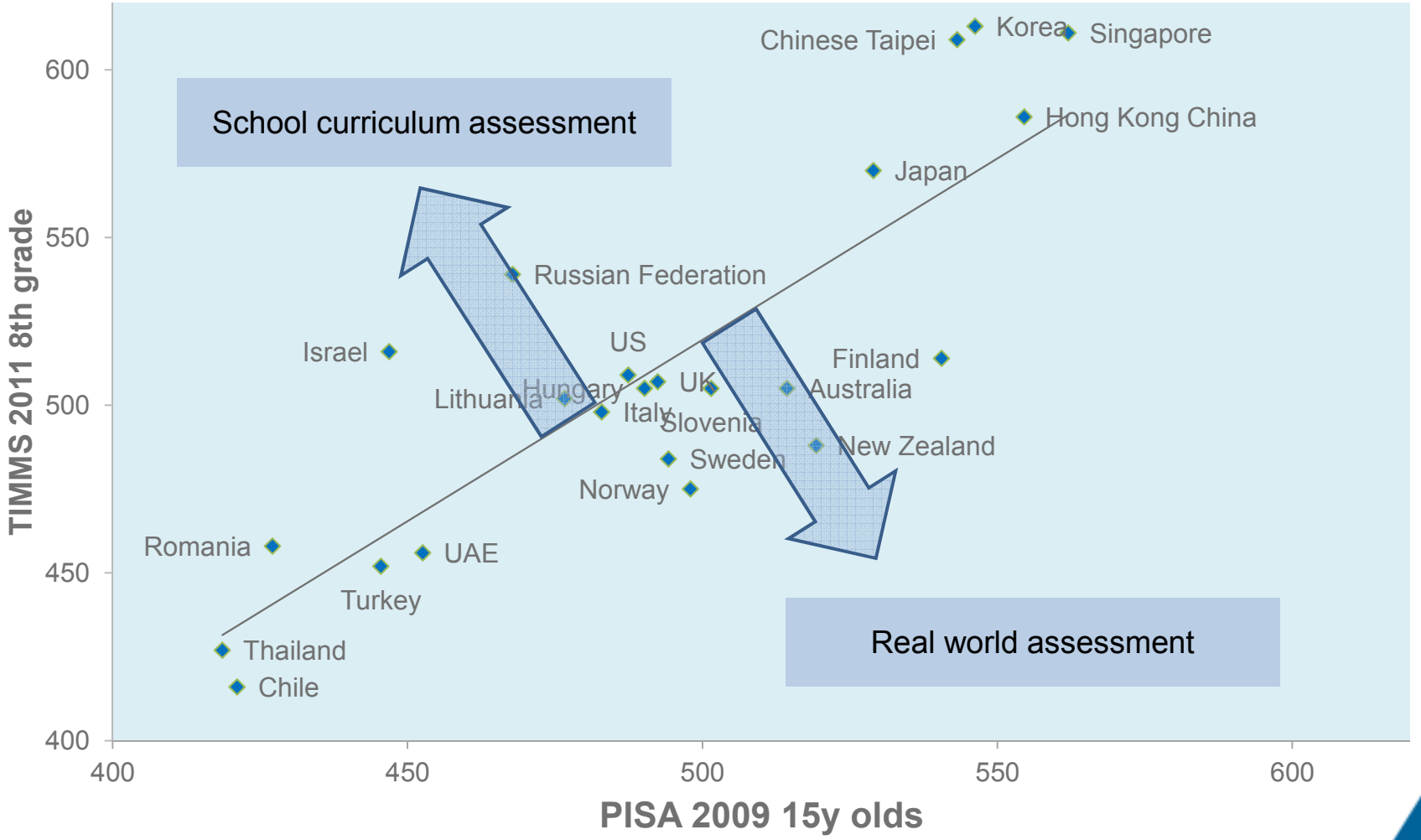

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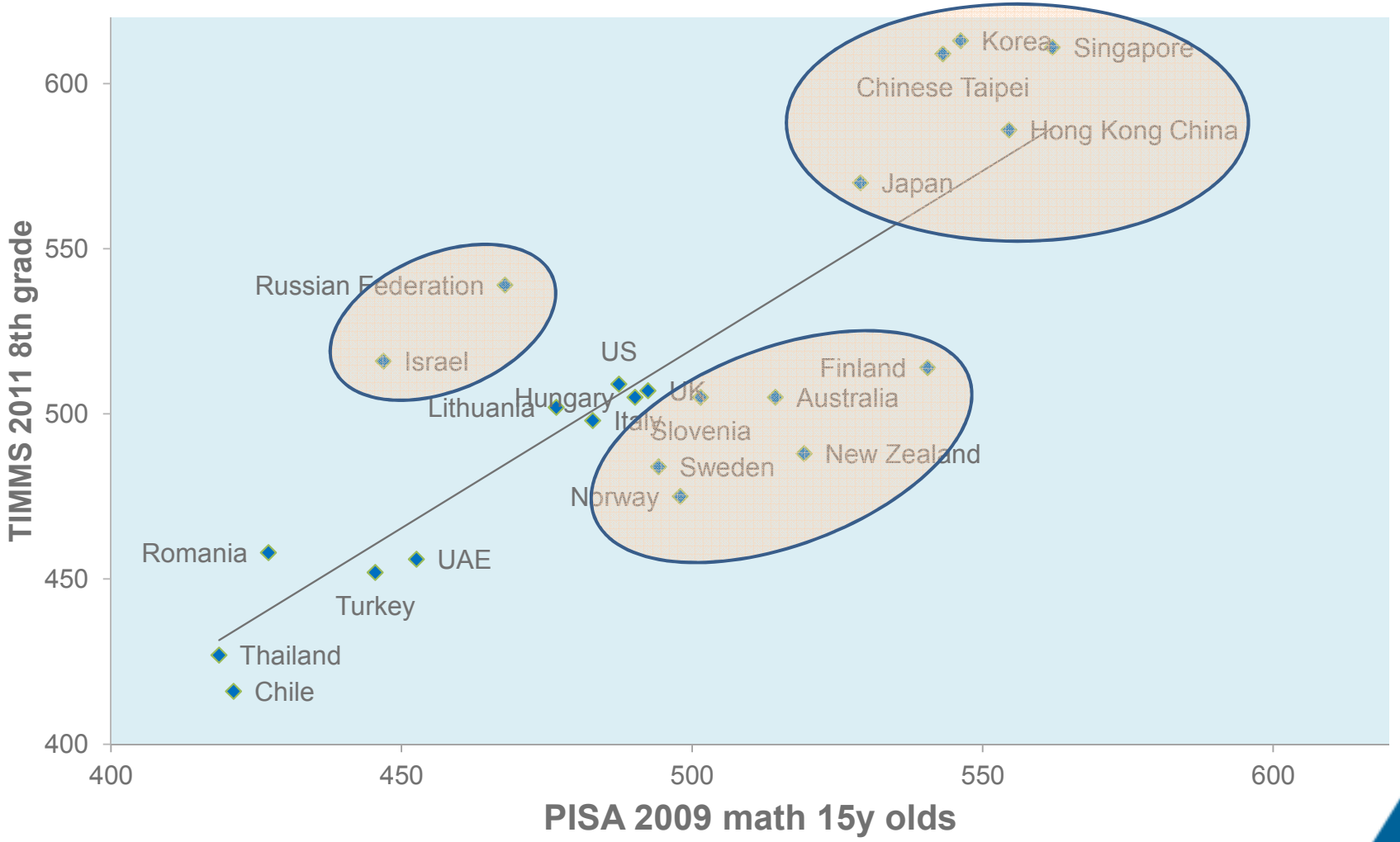

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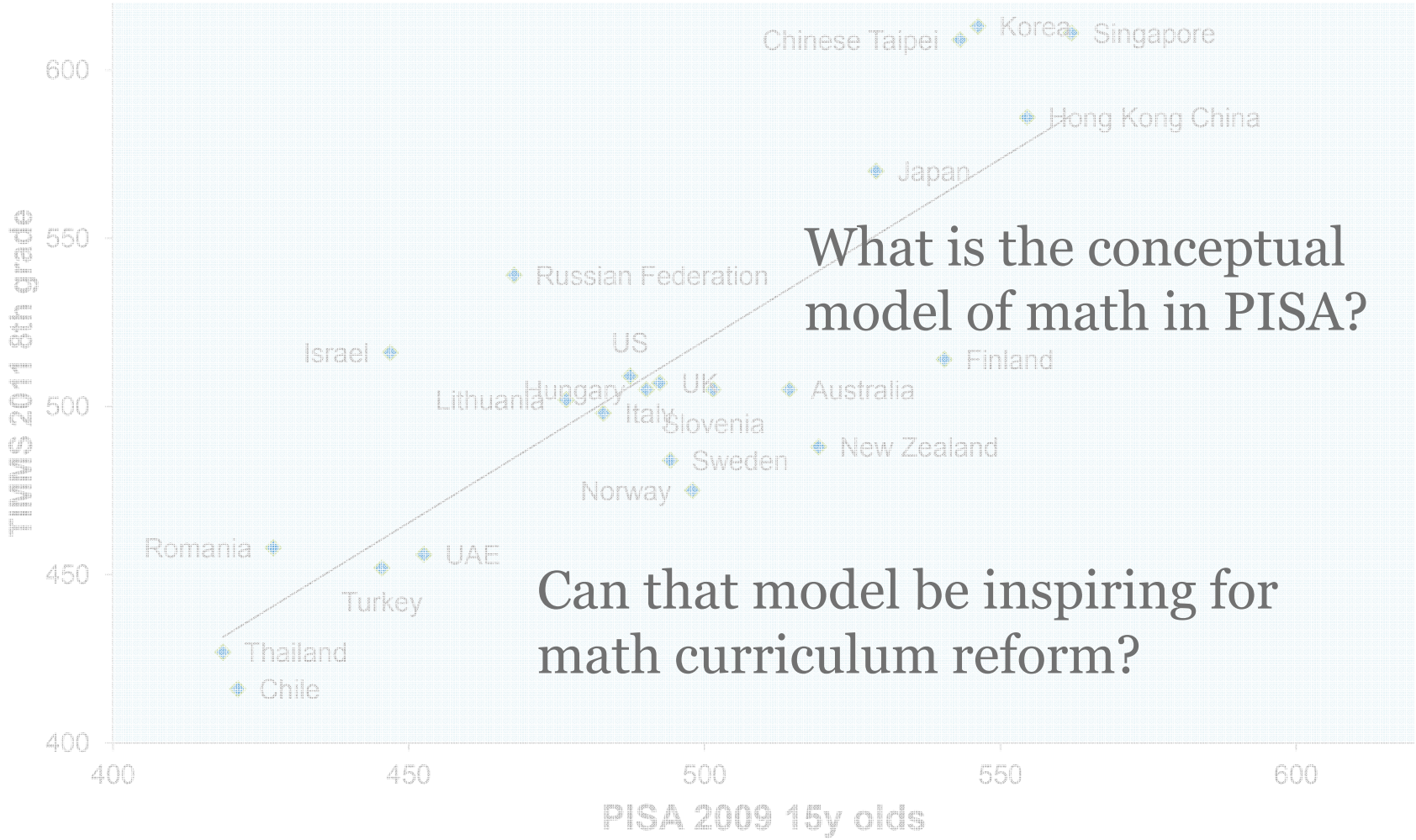
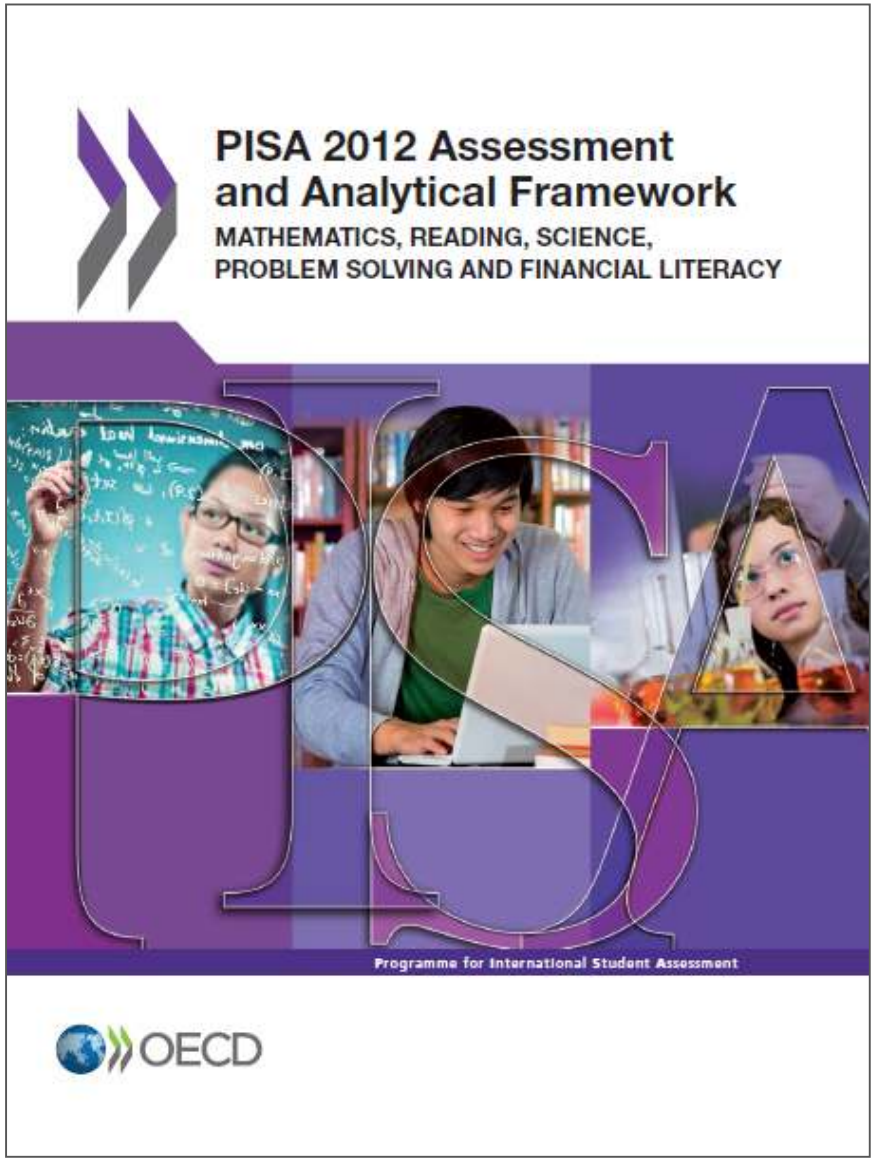


PISA math assessment framework



A definition of mathematical literacy

Mathematical literacy in PISA 2012:

Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.

A model of mathematical literacy

Challenge in real world context

Mathematical content categories: Quantity; Uncertainty and data; Change and relationships; Space and shape

Real world context categories: Personal; Societal; Occupational; Scientific

Formulating situations mathematically...

Involves activities such as:

- identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables;
- recognising mathematical structure (including regularities, relationships, and patterns) in problems or situations;
- simplifying a situation or problem in order to make it amenable to mathematical analysis;
- identifying constraints and assumptions behind any mathematical modelling and simplifications gleaned from the context;
- representing a situation mathematically, using appropriate variables, symbols, diagrams, and standard models;

Formulating situations mathematically...

- representing a problem in a different way, including organising it according to mathematical concepts and making appropriate assumptions;
- understanding and explaining the relationships between the context-specific language of a problem and the symbolic and formal language needed to represent it mathematically;
- translating a problem into mathematical language or a representation;
- recognising aspects of a problem that correspond with known problems or mathematical concepts, facts, or procedures; and
- using technology (such as a spreadsheet or the list facility on a graphing calculator) to portray a mathematical relationship inherent in a contextualised problem.

Employing mathematical concepts, facts, procedures and reasoning…

Involves activities such as:

- devising and implementing strategies for finding mathematical solutions;
- using mathematical tools, including technology, to help find exact or approximate solutions;
- applying mathematical facts, rules, algorithms, and structures when finding solutions;
- manipulating numbers, graphical and statistical data and information, algebraic expressions and equations, and geometric representations;

Employing mathematical concepts, facts, procedures and reasoning…

- making mathematical diagrams, graphs, and constructions and extracting mathematical information from them;
- using and switching between different representations in the process of finding solutions;
- making generalisations based on the results of applying mathematical procedures to find solutions; and
- reflecting on mathematical arguments and explaining and justifying mathematical results.

Interpreting, applying and evaluating mathematical outcomes…

Involves activities such as:

- interpreting a mathematical result back into the real world context;
- evaluating the reasonableness of a mathematical solution in the context of a real-world problem;
- understanding how the real world impacts the outcomes and calculations of a mathematical procedure or model in order to make contextual judgments about how the results should be adjusted or applied;
- explaining why a mathematical result or conclusion does, or does not, make sense given the context of a problem;
- understanding the extent and limits of mathematical concepts and mathematical solutions; and
- critiquing and identifying the limits of the model used to solve a problem

Underlying mathematical capabilities

- Communication
- Mathematising
- Representation
- Reasoning and argument
- Devising strategies for problem solving
- Using symbolic, formal and technical language and operations
- Using mathematical tools

Processes and capabilities

	Formulating situations mathematically	*Employing* mathematical concepts, facts, procedures and reasoning	*Interpreting*, applying and evaluating mathematical outcomes
Representation	Create a mathematical representation of real-world information	Make sense of, relate and use a variety of representations when interacting with a problem	Interpret mathematical outcomes in a variety of formats in relation to a situation or use; compare or evaluate two or more representations in relation to a situation
Reasoning and argument	Explain, defend or provide a justification for the identified or devised representation of a real-world situation	Explain, defend or provide a justification for the processes and procedures used to determine a mathematical result or solution Connect pieces of information to arrive at a mathematical solution, make generalisations or create a multi-step argument	Reflect on mathematical solutions and create explanations and arguments that support, refute or qualify a mathematical solution to a contextualised problem

Processes and capabilities

	Formulating situations mathematically	*Employing* mathematical concepts, facts, procedures and reasoning	*Interpreting*, applying and evaluating mathematical outcomes
Devising strategies for solving problems	Select or devise a plan or strategy to mathematically reframe contextualised problems	Activate effective and sustained control mechanisms across a multi-step procedure leading to a mathematical solution, conclusion, or generalisation	Devise and implement a strategy in order to interpret, evaluate and validate a mathematical solution to a contextualised problem
Using symbolic, formal and technical language and operations	Use appropriate variables, symbols, diagrams and standard models in order to represent a real-world problem using symbolic/formal language	Understand and utilise formal constructs based on definitions, rules and formal systems as well as employing algorithms	Understand the relationship between the context of the problem and representation of the mathematical solution. Use this understanding to help interpret the solution in context and gauge the feasibility and possible limitations of the solution
Using mathematical tools	Use mathematical tools in order to recognise mathematical structures or to portray mathematical relationships	Know about and be able to make appropriate use of various tools that may assist in implementing processes and procedures for determining mathematical solutions	Use mathematical tools to ascertain the reasonableness of a mathematical solution and any limits and constraints on that solution, given the context of the problem

Mathematical ***content*** categories

- PISA uses 4 mathematical content categories:
  - Change and relationships ~ algebra
  - Space and shape ~ geometry
  - Quantity ~ arithmetic
  - Uncertainty and date ~ statistics
- Each category leads to a subscale in PISA
- But individual items can relate to several categories and several categories have to be used to come to an answer

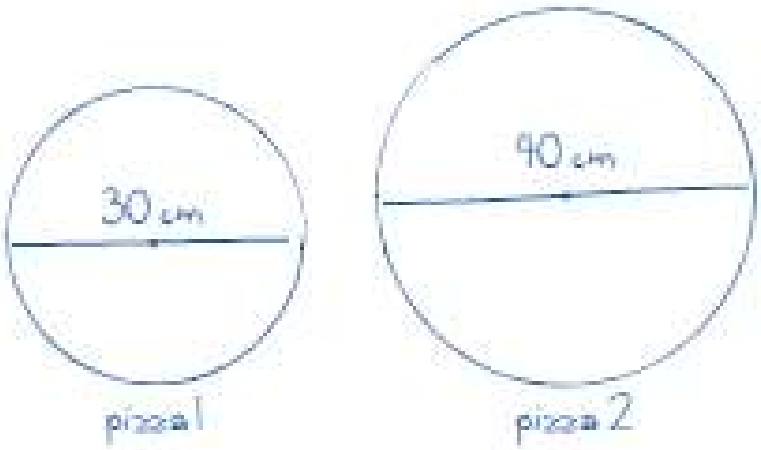
Example item PIZZAS

A pizzeria serves two round pizzas of the same thickness in different sizes. The smaller one has a diameter of 30 cm and costs 30 zeds. The larger one has a diameter of 40 cm and costs 40 zeds.

Which pizza is better value for money? Show your reasoning.

Example item PIZZAS

- Although elements of ‘space and shape’ and ‘quantity’ are needed as well, the item is classified in ‘change and relationships’
- It is placed in the ‘formulating situations mathematically’ process category, although both others are needed as well
- Activation of several capabilities:
  - Communication, mathematising, representation, reasoning, devising strategies



The thickness is the same so I can compare areas.

$$\text{Area of pizza 1} = \pi r^2$$
$$= \pi \times 15 \times 15 \text{ cm}^2$$
$$= 706.5 \text{ cm}^2$$

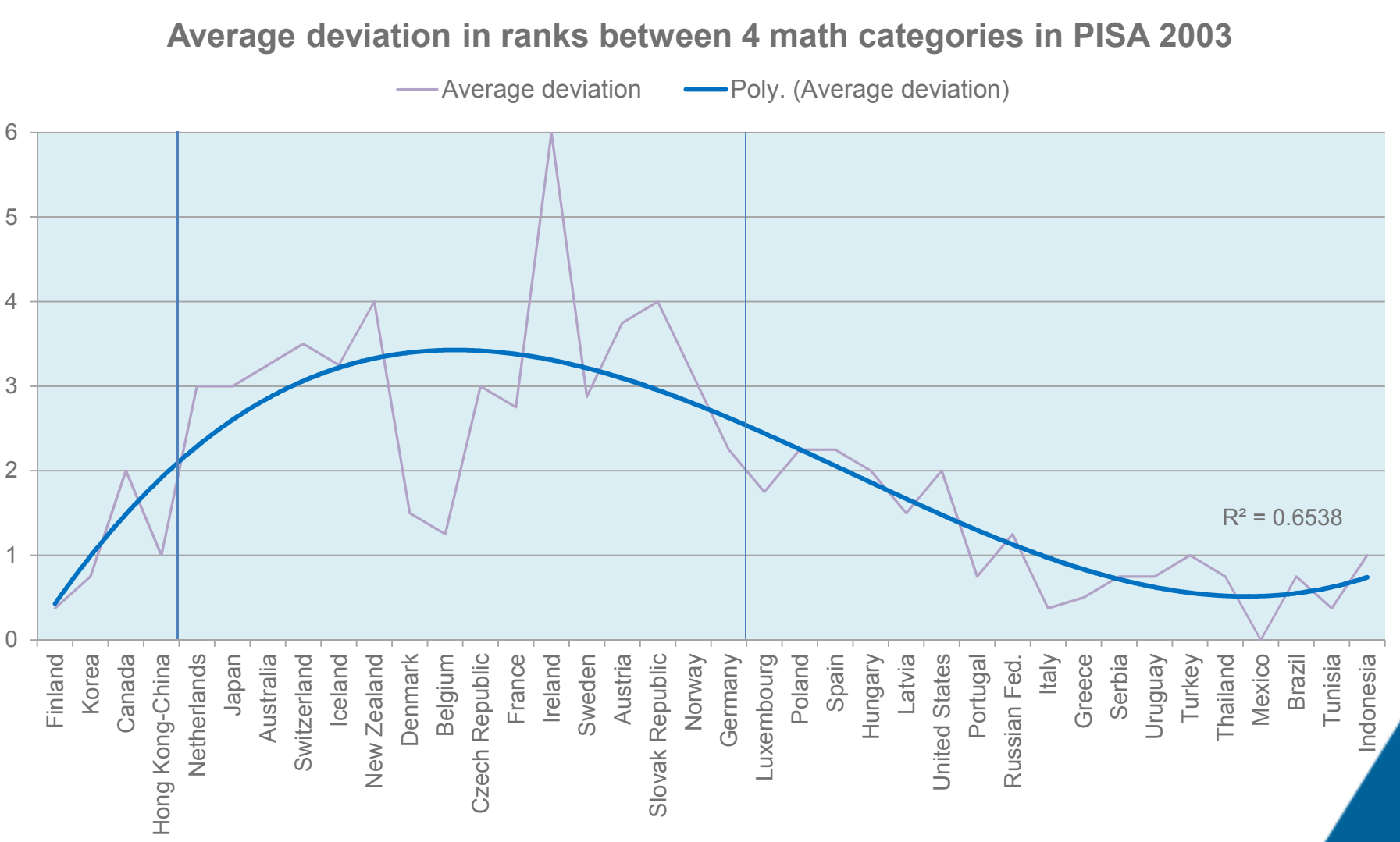
$$\text{Area of pizza 2} = \pi r^2$$
$$= \pi \times 20 \times 20 \text{ cm}^2$$
$$= 1256 \text{ cm}^2$$

$$\text{Cost per cm}^2 \text{ of pizza 1} = 30 \text{ zeds} / 706.5 \text{ cm}^2$$
$$= 0.04 \text{ zeds/cm}^2$$

$$\text{Cost per cm}^2 \text{ of pizza 2} = 40 \text{ zeds} / 1256 \text{ cm}^2$$
$$= 0.03 \text{ zeds/cm}^2$$

So pizza 2 is cheaper per cm^2 + is better value

Coherence in math content categories



Coherence in math content categories

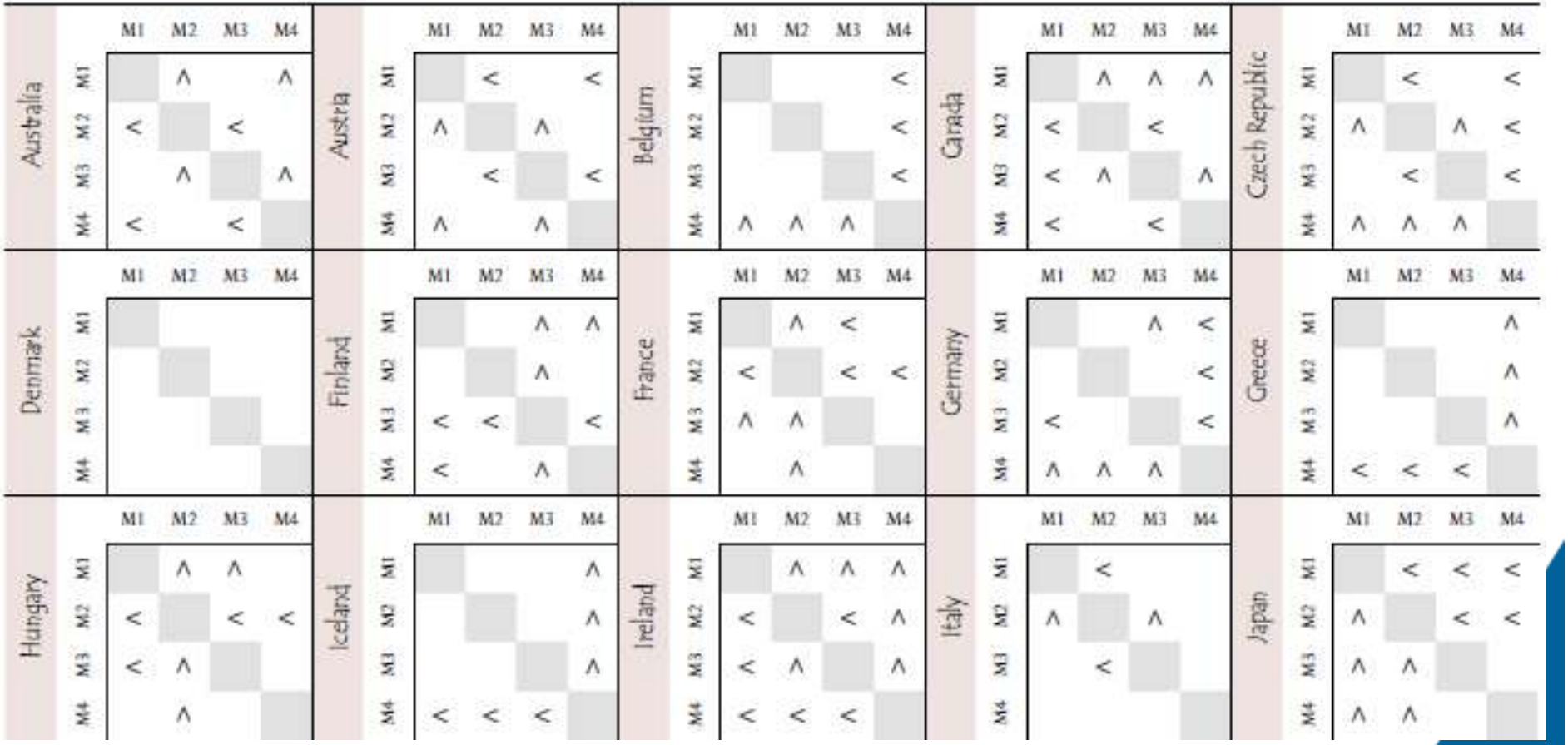
- Coherence has an impact on overall performance:
- At the top (Finland, Korea, Canada) very high coherence
- Increasing lack of coherence causes a country to fall down the ranking from top to sub-top
  - Often only one category responsible for lowering overall performance

Coherence in math content categories

- All of the four content categories are equally responsible for this risk, there is not one specifically responsible
  - But ‘change and relationships’ and ‘uncertainty have slightly more impact
- Coherence between 4 categories improves again below sub-top
  - Overall mediocre performance

Relative performance on the 4 content categories in some countries (PISA 2003)

M1: mathematics/space and shape scale | **M2:** mathematics/change and relationships scale | **M3:** mathematics/quantity scale | **M4:** mathematics/uncertainty scale



shape and space		change and relations		quantity		uncertainty	
1	Finland	2	Finland	1	Finland	1	Finland
4	Korea	3	Korea	2	Korea	4	Korea
9	Canada	4	Canada	6	Canada	3	Canada
2	Hong Kong-China	5	Hong Kong-China	4	Hong Kong-China	5	Hong Kong-China
7	Netherlands	1	Netherlands	8	Netherlands	2	Netherlands
3	Japan	8	Japan	12	Japan	11	Japan
11	Australia	6	Australia	16	Australia	8	Australia
8	Switzerland	15	Switzerland	7	Switzerland	14	Switzerland
14	Iceland	16	Iceland	17	Iceland	7	Iceland
10	New Zealand	9	New Zealand	20	New Zealand	9	New Zealand
13	Denmark	17	Denmark	13	Denmark	13	Denmark
12	Belgium	12	Belgium	14	Belgium	15	Belgium
15	Czech Republic	14	Czech Republic	9	Czech Republic	20	Czech Republic
16	France	10	France	18	France	18	France
26	Ireland	11	Ireland	19	Ireland	10	Ireland
18	Sweden	18	Sweden	10	Sweden	17	Sweden
17	Austria	20	Austria	11	Austria	23	Austria
19	Slovak Republic	22	Slovak Republic	15	Slovak Republic	28	Slovak Republic
25	Norway	23	Norway	25	Norway	16	Norway
20	Germany	19	Germany	21	Germany	26	Germany
21	Luxembourg	26	Luxembourg	22	Luxembourg	24	Luxembourg
22	Poland	27	Poland	23	Poland	19	Poland
24	Spain	28	Spain	26	Spain	21	Spain
27	Hungary	21	Hungary	24	Hungary	22	Hungary
23	Latvia	25	Latvia	27	Latvia	27	Latvia
28	United States	24	United States	29	United States	25	United States
31	Portugal	30	Portugal	31	Portugal	29	Portugal
29	Russian Fed.	29	Russian Fed.	28	Russian Fed.	32	Russian Fed.
30	Italy	31	Italy	30	Italy	30	Italy

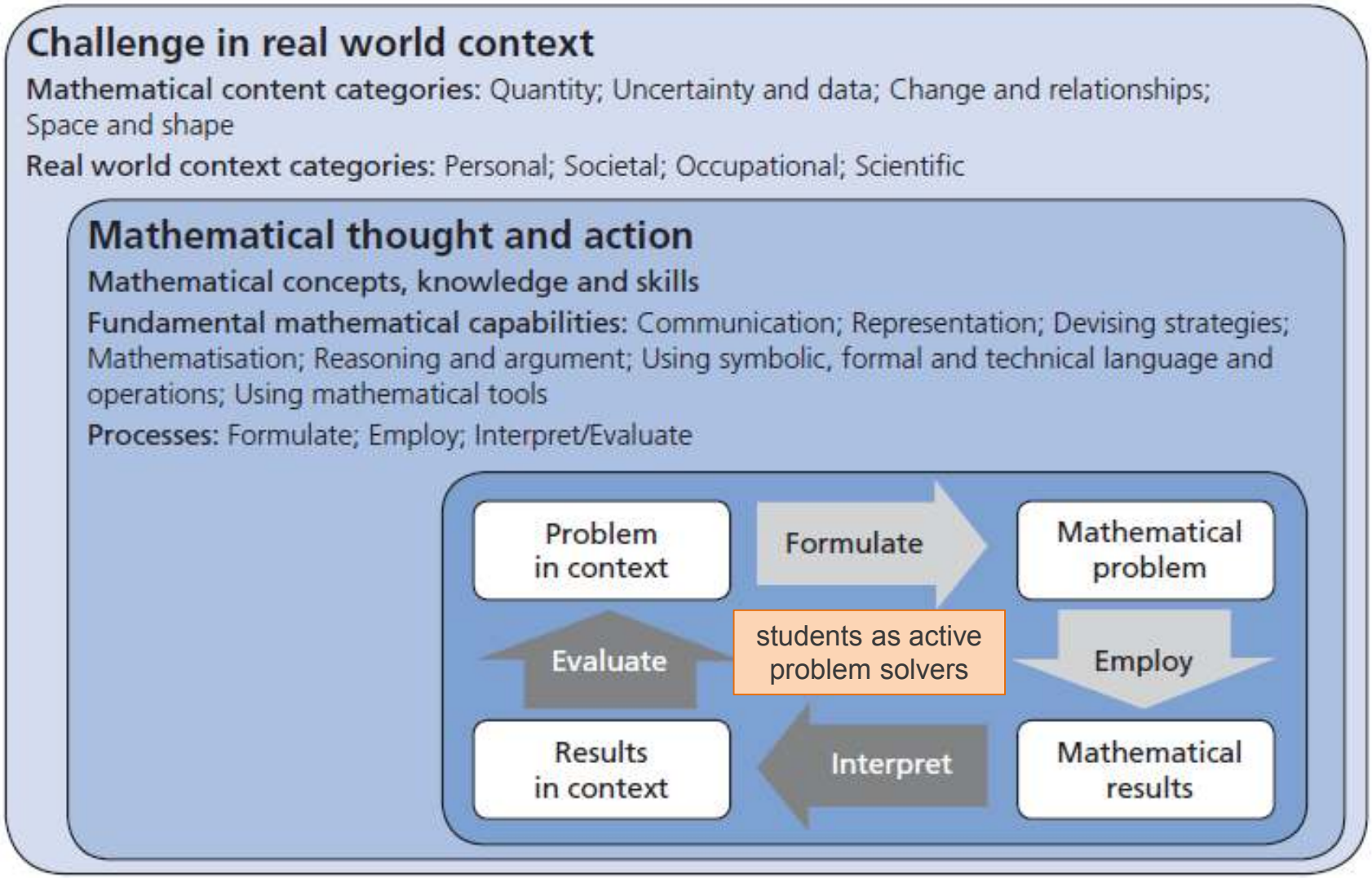
PISA-inspired curriculum reform

- A number of countries have used PISA to inspire curriculum reform in math
  - More integrated
  - More real-world problem-oriented
  - Encouraging making connections
  - More emphasis on mathematical reasoning

Curricular reforms in math

Emphasized in Curriculum	Australia	Alberta, CA	Ontario, CA	Finland	Korea	Shanghai
Equality	●	●	●	●	●	●
Integrate mathematical processes	●	●	●	●	●	●
• Problem solving	●	●	●	●	●	●
• Communication	●	●	●	●		●
• Mathematical Reasoning	●	●	●	●	●	●
Local Issues	●	●	●	●	●	●
Encourage Connections		●		●	●	
Utilize digitally based resources	●	●	●	●	●	●

A model of mathematical literacy



Thank you !

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