

Mathematics and the drift towards purity

or

Why mathematics education is romantic

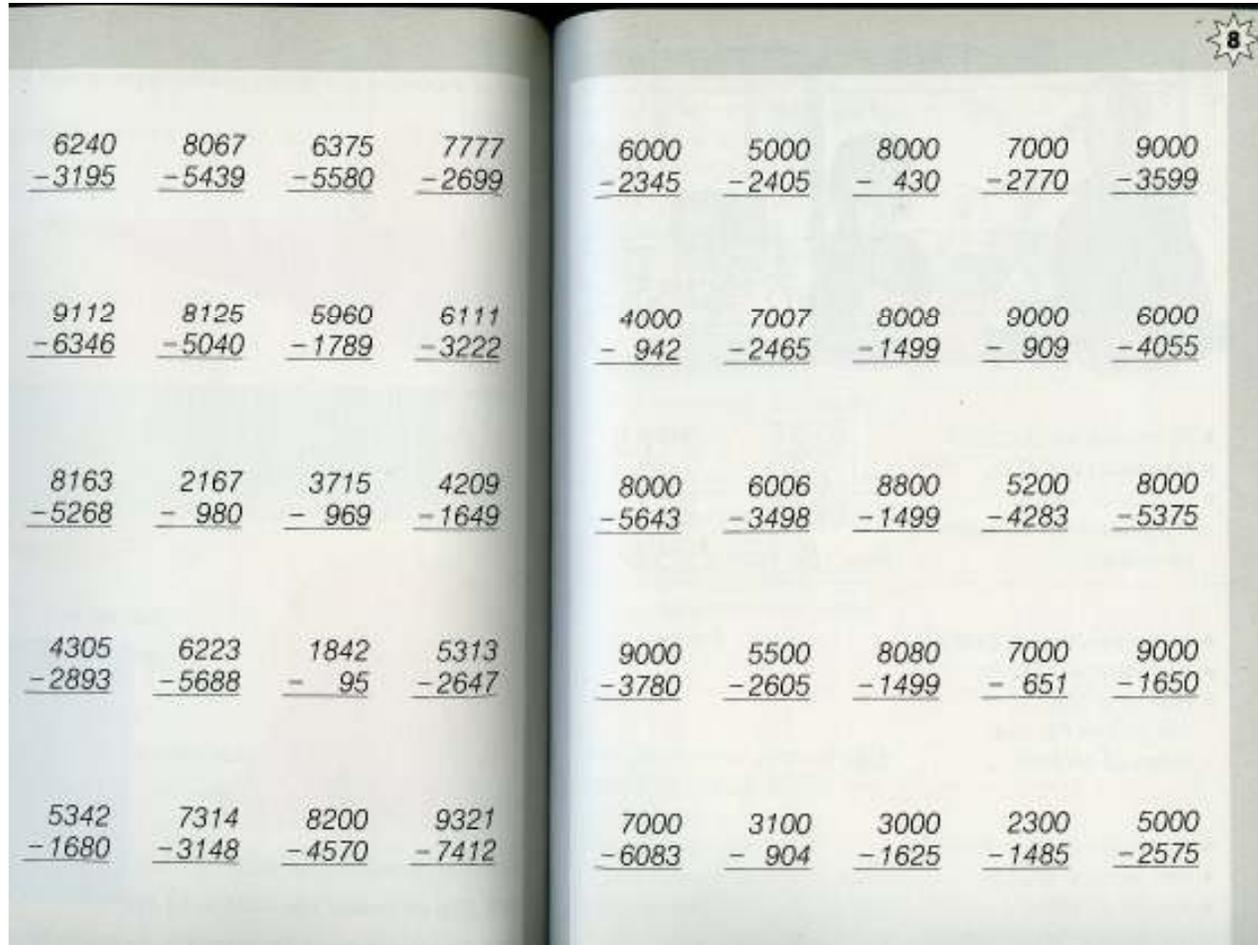
Sverker Lindin

1. A problem
2. A historical explanation
3. Consequences of this history for how we think today about the relationship between learning and technology

- To think mathematically affords a powerful means to understand and control one's social and physical reality.
- Yet despite some 12 or so years of compulsory mathematical education, most children in the developed world leave school with only a limited access to mathematical ideas.
 - Noss & Hoyles, 1996

- As a teacher, you will have the awesome responsibility of helping *all* of your students construct the disposition and knowledge needed to live successfully in a complex and rapidly changing world. To meet the challenges of the twenty-first century, students will especially need *mathematical power* [...]
- Unfortunately, the mathematics curriculum, the instructional practices and assessment methods that you might be accustomed to [...] are generally not effective in promoting children's mathematical power.
 - Baroody och Coslick, 1998

Is this surprising?



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- Arithmetic is universally taught in schools, but almost invariably as the art of mechanical computation only.
- The true significance and the symbolism of the processes employed are concealed from pupil and teacher alike.
- The subtlety, delicacy, and accuracy of mathematical processes have the highest educational value, both direct and indirect.
- To treat them as mechanical routine, not susceptible of explanation or illumination from a higher point of view, is to destroy in large measure the value of mathematics as an educational instrument, and to aid in arresting the mental development of the pupil.
 - David Eugene Smith, *The Teaching of Elementary Mathematics*, 1900

1920s

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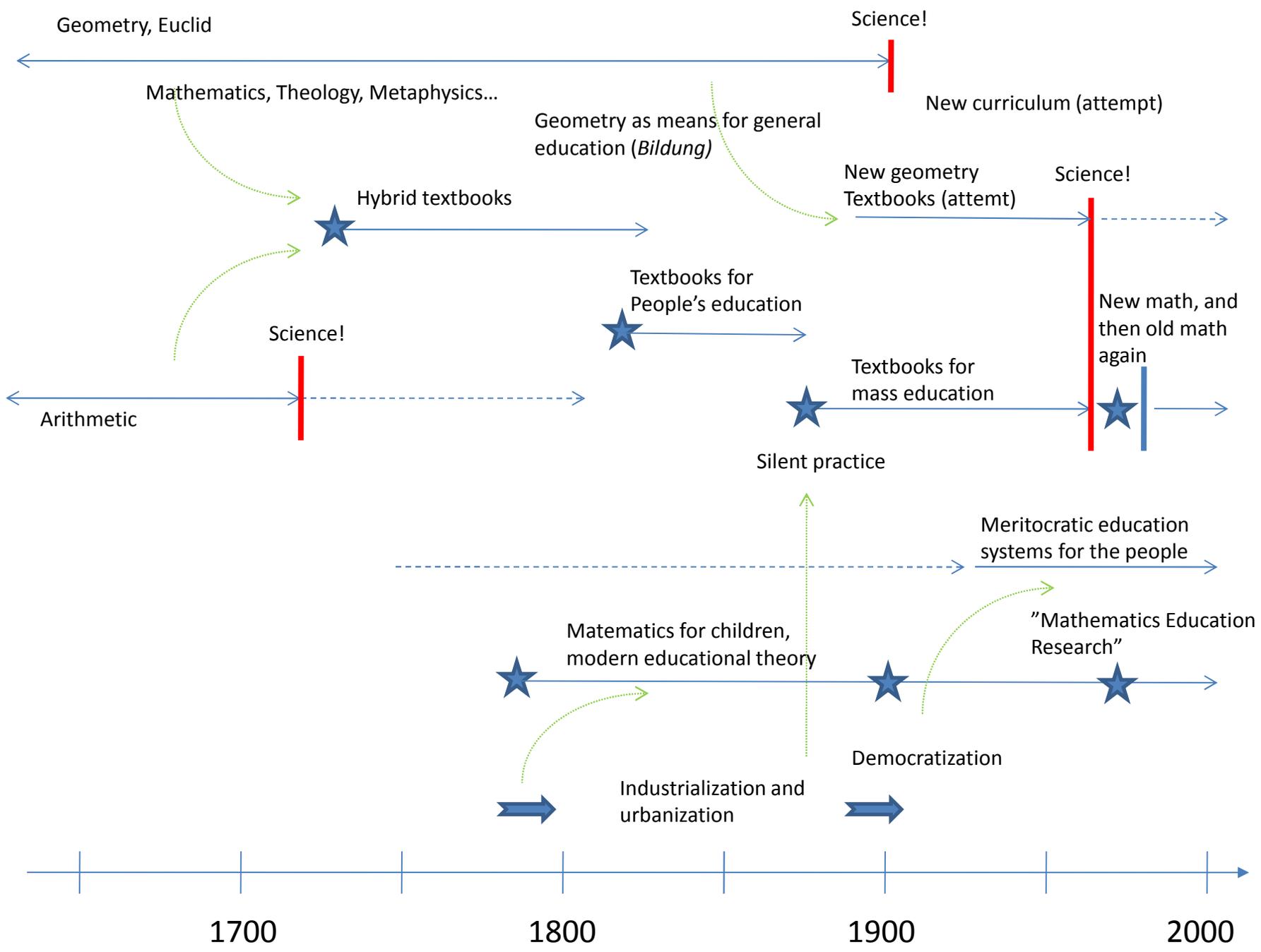

			
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The "Standard Critique"

- Higher goals not directly related to mathematics are to be reached through realistic, relevant, interesting problem solving leading to conceptual understanding.
- Claims that mathematics education how it is actually taught does not lead to these goals. Complaints about mechanism, memorization, rote learning, rule following.
- Calls for reform. Belief that *now* is the time when everything will finally change.
- (But it does not.)

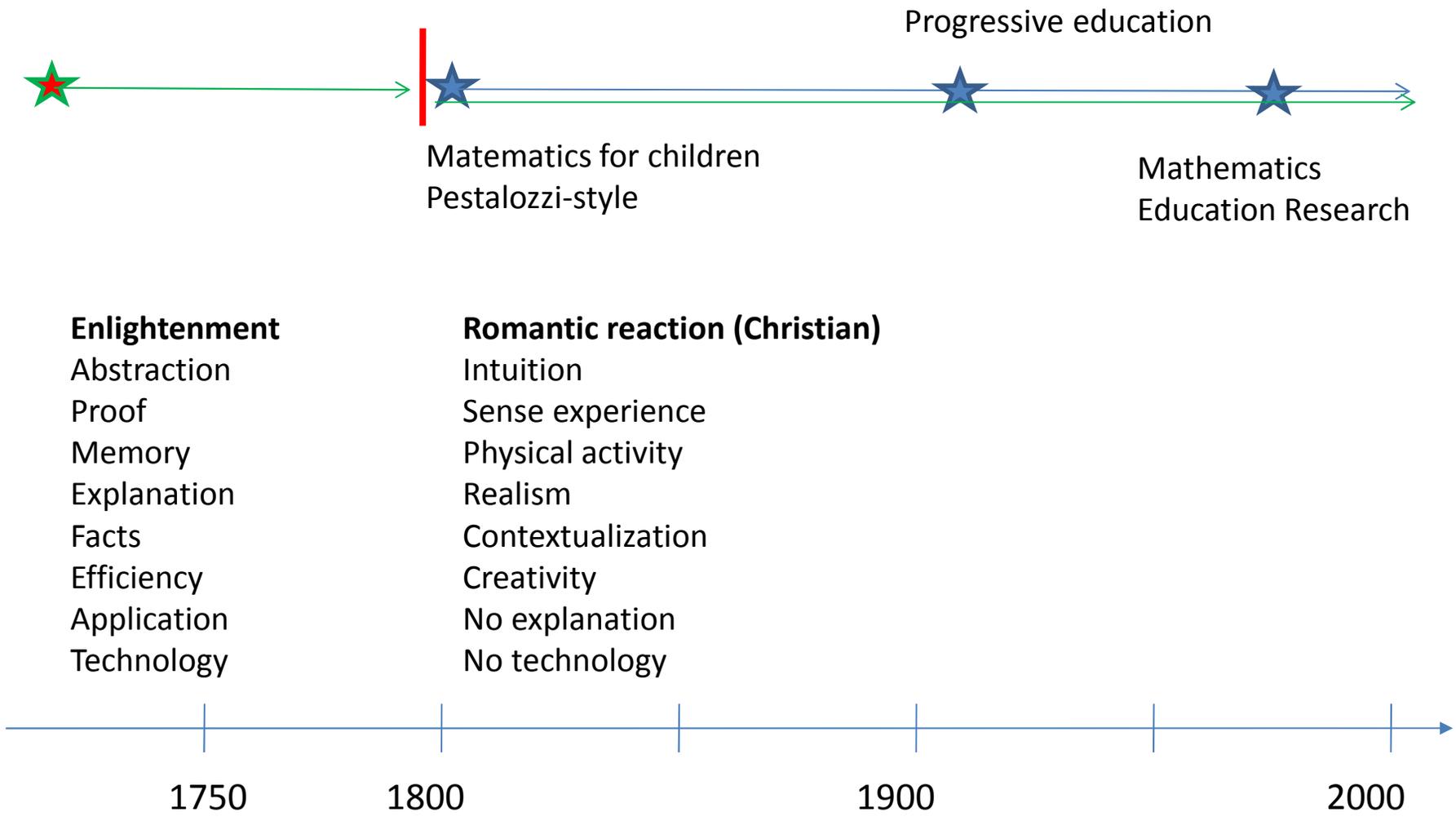
Where does this come from?

Part 2 - history



History is complex

It is necessary to simplify



Mathematics for children
Pestalozzi-style

Progressive education

Mathematics
Education Research

Enlightenment

- Abstraction
- Proof
- Memory
- Explanation
- Facts
- Efficiency
- Application
- Technology

Romantic reaction (Christian)

- Intuition
- Sense experience
- Physical activity
- Realism
- Contextualization
- Creativity
- No explanation
- No technology

1750

1800

1900

2000

- Calculating mechanically is when you follow rules without insight and understanding. Everyone, who does this, is therefore a machine, like a clock, a steam engine or any other machine. But man should not be a machine; she belongs to the organic creatures. To train a child by means of sense-less calculation, by means of play with dead digits, is to dehumanize her, to schackel or kill her spirit. It is an intellectual homicide.
– Adolph Diesterweg, 1830

Split mathematics

Bad for learning	Good for learning
Formalism	Realism
Following rules	Creativity
Memorization	Intuition
Learning facts	Sense experience
Training skills	Physical activity
Being quick	Being slow
Being told, being explained to	No explanation (find out by yourself)
Reading and Listening	No use of tools (struggle is learning)
Use of tools	Understanding

While we perhaps do not "personally" simplify in this way...
the dichotomy is built into the structure of our education systems.
It is reflected in educational theory.
It is expressed in the "standard critique" and sets the direction for reform.

“The drift towards purity”

- Yes, the reaction against enlightenment was about purity! But it is important to understand *what kind* of purity.
- It was a purity in *separation* from modern scientific and technological society; from efficiency and instrumentality.
- The vision was that work with mathematics would put the child into a harmonic, “pure”, relationship with her environment. For this, “realism” was crucial – both for understanding the presence of mathematics in the world, and for understanding the usefulness of mathematical knowledge.
- The central idea was that mathematical knowledge **must** come from the inside, it must be discovered, grown, built. This made the activity of *problem solving* central for learning. Tools and explanations was considered to distract from this process.

What does this mean today?
Part 3 - technology

A history of reluctance

New technology	Preferred in mathematics education
Pen and paper	Mental arithmetic
Decimals	General fractions
Algebra and equations	Intuitive methods
Pocket calculators	Calculation by hand
Computers	Calculation by hand

- From the perspective of modern mathematics education, tools and technology seems to constitute an obstacle to learning, because it removes the "problem" from many of the problems solved in school.
- But the standard critique indicates that the work-without-tools that pupils perform in school look very different depending on perspective:
 - In the past and present it usually looks like calculation
 - Only in the future - as envisioned! – does it look like interesting, fun, relevant problem solving leading to useful knowledge.

Conclusions?

1. Be careful with the higher goals. (democracy, thinking)
2. Think about *how* what students actually *do* in school, could leads to these goals. Expect no magic from the subject matter of mathematics.
3. Open up for other ways of approaching mathematics than through tool-free, "pure", problem solving:
 1. By for instance reading about its history and sociology - its relationships to theology and technology alike - and discussing it. This would make mathematics education *educating* in a way that problem solving can never be.
 2. By allowing the students to use proper tools when working with problem solving.
4. Recognize that we have perhaps not only inherited a *problem* from history, but perhaps also, less visibly, its "*solution*". We need to rethink both.