Introductory Statistics Learning Outcomes

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CENTER FOR CURRICULUM REDESIGN

About the Center for Curriculum Design

The Center for Curriculum Redesign (CCR) is a non-profit global organization dedicated to improving Education via answering the question: **"*What* should students learn for the 21st century?"** CCR brings together international organizations, jurisdictions, academic institutions, corporations, and non-profit organizations including foundations. It focuses on both designing and propagating new curricula.

Project Team

Charles Fadel, Founder, Center for Curriculum Redesign

Charles Fadel is a global education thought leader and author, futurist, inventor, founder and chairman of Center for Curriculum Redesign, project director Harvard Graduate School of Education, member President's Council at Olin College of Engineering, education committee chair at BIAC/OECD, and co-author of "Artificial Intelligence in Education" (2019), "Four-Dimensional Education" (framework in 21 languages), and best-selling "21st Century Skills."

Eve Howard, Project Manager

Eve Howard has extensive experience in post-secondary curriculum design and development, and editorial and academic product management across the humanities, business, and STEM disciplines. Before her work with CCR, Eve was VP Editor-in-Chief for Cengage Learning.

Paul Thomas, Team Lead

Paul Thomas has spent the past 25 years developing research-driven curriculum for small startups, public corporations, and nonprofits. His work includes dozens of courses for virtual, blended, and traditional classroom contexts as well as textbooks. Paul began his career as a high school math and computer science teacher and holds degrees in Mathematics and Operations Research from George Mason University.

Acknowledgments

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Lead Author

Beverly Wood

ASA Section on Statistics & Data Science Education; SIGMAA Statistics & Data Science Education; Associate Professor, Embry-Riddle Aeronautical University

In-Depth Reviewers

Prince Afriyie, University of Virginia

JD Lopez, Univ of Arizona

Roxy Peck, Cal Poly, San Luis Obispo

Nicholas Horton, Amherst College

Beth Chance, Cal Poly, SLO

Aris Winger, Georgia Gwinnett College

Association Supporters

Authors of GAISE 2016*

Megan Mocko, Co-Chair, University of Florida

Michelle Everson, Co-Chair, Ohio State University

Jeff Witmer, Oberlin College and Conservatory

Beverly Wood, Embry-Riddle Aeronautical University

ASA, Section Statistics and Data Science Education*

Nathan Tintle, ISI, Statistics

Ann Cannon, Cornell College

Jo Hardin, Pomona College

AMATYC Statistics ANet

Rebecca Wong, West Valley College

*Please note that permission to include these thought-leaders is currently in progress. This list is for BMGF personnel and partners. (4.23.22)

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Introduction and Executive Summary

Project Goals and Definition of Learning Outcomes

Our goals are to benchmark and develop a set of faculty and discipline-association aligned and equity-centered learning outcomes for Introductory Statistics. For the purposes of this work, we define learning outcomes as **measurable student performance expectations based upon what the student learned in each core topic area**.

Project Approach

This work represents a codified process and scalable approach to developing faculty and discipline-association aligned learning outcomes that function as measurable instructional goals for faculty teaching these courses; domain associations endeavoring to codify essential undergraduate domain knowledge; courseware providers developing rich, interactive, and adaptive curriculum resources to support instruction and assessment in gateway courses; and students as consumers and buyers of college credits to better understand the core knowledge and competencies they should gain by the completion of Introductory Statistics. Course curriculum is defined as the organizing principles, essential and factual content, and procedures that constitute the knowledge base of a domain. Quality curriculum includes structured learning activities that foster problem-solving and inquiry enabling students to both encode new knowledge based on prior knowledge and *transfer* new knowledge into additional contexts.

The Center for Curriculum Redesign (CCR) is a Boston-based international non-profit research and "education engineering" organization working with educators, researchers, organizations, associations, and thought-leaders to define a 21st-century curriculum for students in K-12 and Higher Education. CCR was selected by the Bill and Melinda Gates Foundation to lead this demonstration project because:

1. Global reputation: CCR was founded by Chairman/CEO Charles Fadel in 2012 with the goal of reshaping the “What” of Education *in the context of social and economic justice/equity*, which are being amplified by new technologies (AI, Biotech, etc.) and planetary-wide problems (global warming, pandemic, etc.).

CCR has extensive experience managing large-scale curriculum redesign and learning outcomes development projects for global organizations such as the Organization for Economic Cooperation and Development (OECD). Its organizational focus on K-12 and Higher Ed, using innovative approaches based on science - and evidence-based, precise, and diligent processes have led CCR to become *the only strategic partner to the OECD's Education Directorate (PreK-20)*, with a decade-long relationship. CCR has a decade-long track record of developing equity-minded and social justice-centered policy recommendations, tools, resources, methodologies, and frameworks for making education more relevant for the 21st century and to better prepare all students - particularly historically marginalized students - for life and work in a rapidly changing global economy.

2. CCR is *the* leading organization in Competencies - aka "21st Century Skills" and "Social-Emotional Learning" (SEL) - development, creating a sophisticated yet actionable competencies/sub-competencies framework to include 21st-century workforce skills in addition to "Knowledge" and to describe precise implementation at the instructor level as well as formative assessments.
3. Equity is a centerpiece of this framework, as there are *explicit intersections* between the sub-competencies and elements of social justice.
4. Curriculum expertise: CCR is presently developing a new test called the "Primo" option for the OECD based on modern mathematics encompassing six major topics (stats/probability, Bayesian probability, exponentials, algorithms, complex systems, game theory).
5. Technology tools for deeper human analysis: CCR has made ongoing investments in technology tools to help synthesize and speed up the comprehension of deep structures in Knowledge and Competencies and their deployment to include analyzing the competency employment needs of industry. CCR's pre-authoring environment will help courseware developers effectively develop a range of formative assessment types aligned to learning outcomes and access a robust equity-enabling *instructional strategies* database to incorporate key instructional tools into the curriculum to support faculty professional development for outcome-driven course design and delivery.
6. One key ongoing project CCR is heading is to modernize the OECD's global PISA test, by incorporating high-impact-but-low-algebraic-complexity

mathematics, such as probability and statistics and discrete and computational math.

CCR combines a rigorous methodology which includes collection and analysis of multiple points of data as well as collaborations that encourage creativity to break down boundaries and highlight salient innovations for inclusion in a modern curriculum in the various disciplines.

Higher Education Gateway Courses

Each year, more than three million students enroll in roughly 20 general education courses in U.S. higher education. According to research from the National Center for Academic Transformation (NCAT), just 25 courses generate roughly half of all student enrollments in community colleges and about a third of enrollments in four-year institutions. Successful completion of these courses is key to student progress toward a quality degree or credential. The Gardner Institute has identified these “gateway courses” as foundational, credit-bearing, lower-division courses, for which large numbers of students are at risk of failure, and thus stand as “gatekeepers” to degree completion.

For this project, CCR’s charter is to undertake a demonstration project to define the measurable, specific learning outcomes for four key undergraduate gateway courses, and for which successful completion is critical for college success. These four courses are Introductory Statistics, General Chemistry, Introductory Psychology, and Introductory Sociology.

For the first demonstration project, CCR focused on Introductory Statistics, due to the following factors:

- Presents compelling opportunities for complementary exemplars with potential to impact our focus students while also driving market innovation and setting a new standard for excellence in the market.
- Has high annual enrollments and high drop-fail-withdraw-incomplete (DFWI) rates (24% DFWI rate), making Introductory Statistics an excellent candidate for new, innovative, and more effective courseware. Targeting high enrollment courses

also increases the potential to recruit sufficient populations of students from our priority populations to enable rigorous research design.

- Is increasingly offered as a requirement in its related domain of study and subsequent STEM pathways. For example, enrollment in Introductory Statistics is growing rapidly due to its adoption as a pillar of both Math Pathway & Guided Pathway reforms.
- Has strong courseware adoption rates by faculty (43%), greatly simplifying the challenge for developers to secure pilot sites. Faculty place relatively strong value on evidence-based teaching, providing a good testbed for validating assumptions about how courseware can best enable and support evidence-based teaching practices.
- Offers opportunities to test various content and technical approaches to improving student learning outcomes such as eliminating knowledge gaps and engaging students with real-world problems and data sets, while also allowing for opportunities to measure and assess shifts and improvements in content.
- Has synergies with exemplars of excellence in features and function equity-centered courseware that addresses gaps in availability, curriculum alignment between learning outcomes, activities, and assessments.

To advance the work, CCR worked with a team of experienced faculty currently teaching Introductory Statistics. Additionally, equity experts in the field of statistics were commissioned to guide the development of course learning outcomes, discipline core concepts, key missing topics, and profound examples to enliven the concepts and equity focus of the work.

The lead subject matter expert for the Statistics Learning Outcomes is Dr. Beverly Wood. Dr. Wood teaches Introductory Statistics at Embry-Riddle Aeronautical University and is an active member of the American Statistical Association, and co-author for the Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report, 2016. Prior to Embry-Riddle, she taught at two-year institutions and a satellite campus catering to working adults earning business degrees.

About General Statistics

The role of data in everyday life and in professional decision-making continues to grow in importance for all citizens. The introductory statistics course may be selected by students to fulfill a general education requirement, co-requisite requirement, as a prerequisite to a discipline specific-second course, or to simply explore and discover a prospective career interest. Our work as follows is built upon the diligent and insightful work of others; most notably the American Statistical Association's GAISE College Report and the University of Texas at Austin's Dana Center Mathematics Pathways.

Feedback

Please send any feedback on this document to HigherEdLOs@curriculumredesign.org.

Key Understandings

The following pages present tables of Learning Outcomes (LOs) logically grouped by Introductory Statistics anchoring Topics rather than a linear sequence suggesting a single way to move through the material in a classroom. The conceptual ideas in each Topic are woven together with ideas in other Topics. How intricately or loosely that is done depends on student preparedness and classroom environment (meeting schedule, class size, available technology, etc.). Topics are ordered in a way that makes sense, but the full list of LOs are not entirely ordered in a sequence that is perfectly ideal. For example, the concept of standard deviation is not only a part of "Use technology to investigate data…" where it will be introduced through its computation and usefulness in interpreting data displays, but also fundamental for the general understanding of variability that is in many subsequent LOs. A course developer could design a course that progresses through these Topics in order, however it is quite reasonable to create other sequences that cover all the same Statistics Topics and LOs.

Our work began with canvassing Statistics curriculum recommendations from authoritative sources such as the discipline associations, innovative teaching centers and faculty. This search led to the American Statistical Association (ASA), the leading scholarly association and the Charles A Dana Center at UT Austin, a revered group focusing on equity-centered courseware and professional development supporting math reform. We compared recommendations from the American Statistical Association (ASA) and the Dana Center Mathematics Pathways (DCMP), then merged them into our first draft LOs. Further comparison was done with the content of leading academic textbooks such as Triola, Elementary Statistics, 14th ed, 2022 and the College Board's AP Statistics course; The CAOS (Comprehensive Assessment of Outcomes in Statistics) Exam, developed by the Assessment Resource Tools for Improving Statistical Thinking project; and recommended "cutting edge" textbooks. After analyzing and documenting the concepts covered in these various curriculums, as well as tapping into our previous statistics work and review of current statistical applications, our experts identified topics that were not adequately represented in the LOs for both equity and modernity. As a result of our analysis, the LOs (highlighted in blue below) were added to address equity considerations and/or modern demands on the content coverage of statistical concepts, topics, and outcomes.

To achieve our equity goal of ensuring that the Statistics LOs reflected the diverse racial, ethnic, and linguistic make-up of today's higher education students and provide a culturally representative experience for all students, we specifically examined and, in many cases, reframed the LOs using an equity and social justice lens. Examples of reframing LOs include ensuring that LOs:

- provide opportunities and examples for students to connect what they are learning to their lived experience
- are not limited to or reinforcing the ideals, philosophies, or perspectives of one dominant culture
- include opportunities to highlight stereotypes commonly found in the discipline like awareness of sampling sizes and causality as well as bias in interpretation.

Equity considerations also impacted our recommendations about the use of technology in Introductory Statistics. While some of these considerations fall outside the actual development of LOs, we endeavored to capture these critical ideas to help faculty, course designers, and courseware developers to design learning environments that provide equity and culturally centered learning experiences for all students. You'll see these notes in sections of the work labeled "Examples, Pedagogy, and Technology."

Since the goals of an education are both expertise *AND* transfer, our work has focused on paying deep attention to four facets, which need to be intertwined and "braided" together during courseware development:

- Essential Content: These are the LOs, expanded to include the equity and modernization aspects discussed above.
- Core Concepts: They represent the epistemological lens that experts apply when looking at the world (see further description below).
- Interdisciplinarity: The use of examples from different disciplines (law, sociology, etc.) ensures relevance.
- Competencies: Life and work require the mastery of "21st Century Skills/Social-Emotional Learning," yet precious little is done in education to insure their explicit identification and their learning - *deliberately, systematically, comprehensively, demonstrably.*

Our work provides this guidance to the course developer, by explicitly identifying the relevant (top 4, mid 4, bottom 4) competencies for Mathematics:

	Competencies											
Discipline	Skills				Character						Meta-Learning	
	Creativity	Critical thinking	Communication	Collaboration	Mindfulness	Curiosity	Courage	Resilience	Ethics	Leadership	Metacognition	Growth Mindset
Mathematics		Core							Data bias		Core	

It is the *combination* of all these aspects - content, concepts, interdisciplinarity (examples), competencies, and pedagogy (including technology) - that not only reinforces knowledge creation but *ensures knowledge transfer*.

Core Concepts

Core Concepts are abstract principles that can be used to organize broad areas of knowledge for a given domain, make inferences within a domain, and solve a wide range of problems. Each Core Concept describes a lens that experts use to see their domain and the world. LOs convey what should be learned, while Core Concepts describe the "so what."

The set of Core Concepts creates a scaffold that helps students develop meaningful connections which lead to deeper real-world understanding and more robust ability to solve problems. They are an essential piece to transfer.

Definition of Core and Stretch LOs

LOs are presented in two columns labeled "Core" and "Stretch."

The primary **use case** for Core Learning Outcomes is to support a one-semester, introductory, non-calculus-based college course in statistics. For many students, this statistics course may be their only course of study in the domain.

Stretch Learning Outcomes require more complex math and can be included in general statistics courses that provide the foundation for further study in math and sciences.

The column headings include an approximate indication of time it may take to cover all the LOs within the Topic. This is not meant to suggest a precise number of hours equally spread across the LOs within that Topic. The table column “Time on Task” gives general guidance about the priority and time commitment at the Core level, legend below.

A = priority, substantial part of the class period
B = priority, notable amount of time in the class period
C = cover, doesn't take notable amount of time
D1 = optional coverage, but it will take a notable amount of time
D2 = optional coverage, won't take much time.
Spiral = not a once-and-done concept, needs revisiting at various other points in the curriculum

The Topic titles preceded by ***** contain LOs that should deliberately spiral throughout the course. Again, items within a Topic may not be addressed collectively or even consecutively and will probably be addressed multiple times across the course.

Technology

It is essential to focus on interpretation of the statistical topics. Moving the mathematics to a computer or software tools frees up the time to do so in a single semester. This impacts equitable education by removing barriers of computational skill as well as enhancing the technical proficiency necessary in many workplaces that require post-secondary education.

Types of software include dedicated statistical software (e.g., SAS, SPSS, or Stata), general mathematical software (e.g., MATLAB or Mathematica), spreadsheet tools (e.g., Excel or Sheets), and even programming languages (e.g., R, Python). Any of these types of tools could be used for any or all the statistical computations essential for this course.

- To achieve mastery of Core LOs, web-based applications that support basic calculations are sufficient
- To achieve mastery of Stretch LOs, the selection of technology tools will likely depend on faculty expertise and the resources available to them and their students.

Concerns about student access to technology is by no means a small consideration. Whatever the constraints, *some* exposure to the tools used by employers (even of modest size) is an important component of statistical education. At the very least, output from more than one software should be provided from which there should be practice at extracting understanding and interpreting results. The type of software used by the student or presented by the instructor depends on institutional access, of course. See the "Suggested Resource Material" list at the end of this document for some places to start a web search for examples for software other than what is available on your campus. A robust community of statistics educators willing to help instructors with limited resources is also listed on the final page.
What about graphing calculators? Unless there is totally inadequate access to the internet (even if it is only by smartphone), the purchase of a calculator is an unnecessary financial burden for students and an antiquated mindset. Smartphones can run apps and access websites that obviate the need for a separate calculator device.

Prerequisites

The few mathematical prerequisites are listed in the leftmost column of each table. In general, they are skills from secondary schooling. In courseware development these prerequisites could form the basis for unit-level readiness assessments, supports, and/or resources to help faculty assess student readiness and bridge individual gaps.

A Supporting Topic section is dedicated to Probability. The use of technology for handling descriptive and inferential statistics mitigates the need for deep understanding of the mathematical roots of statistics for the casual users these students tend to become. However, probabilistic reasoning is a skill all students (not to mention already-educated decision-makers!) need to develop, as it pertains to everyday decisions in real life - sometimes literally life-or-death! Some suggestions for integrating probability into this course:

- Incorporate into quantitative literacy/reasoning courses
- Provide supplemental instruction
- 1-credit co-requisite (lab)
- 4-credit course

Pedagogical Remarks

Reviewers provided useful feedback to strengthen this set of LOs. Many of the remarks also include some ideas about the pedagogy, especially regarding equity approaches. This information is in the rightmost column.

Introduction to Statistics Learning Outcomes

The LOs described in this document are also visible in a spreadsheet available at https://curriculumredesign.org/wp-content/uploads/CCR-LOs-Intro-Statistics-protected.xlsx

TOPIC 1: Questions that can be answered by data analysis

There are numbers being used to influence decisions everywhere! The learnings in this topic should lead to an understanding that those influential numbers come from someplace, by someone, and for some reason. Balancing skepticism of all numbers with faith in every number begins with an understanding of what data can and cannot offer to the decision-making process. This may be introductory information but is likely to be revisited through other topics.

Core Concepts: Operationalizing Our Question, Sampling, Data Distributions, Summarizing Data, Features of Visualization

Prerequisite mathematics	Time on Task	CORE LOs (5%)	STRETCH LOs	Pedagogical remarks from reviewers
	C Spiral	Explain the value of generalizable information over individual experience in decision-making	Explain the value of generalizable information over individual experience in decision-making; introduce concerns of validity	"Data beat anecdotes." There is also a tradeoff between more homogenous observational units and generalization
	C	Distinguish between questions that are answerable through descriptive statistics, inferential statistics, or neither	Distinguish between questions that are answerable through descriptive statistics, inferential statistics, or neither	

	B	Distinguish between a census and a sample, taking care to not suggest that sample size is the distinguishing feature	Distinguish between a census and a sample, taking care to not suggest that sample size is the distinguishing feature	Stats should better define "population." Note that it doesn't make any sense to think about inferences for census data since we know the population values; might include the idea of "random process" as a population under SBI approach
	C Spiral	Recognize that statistical inference involves drawing conclusions about a population based on data from a sample	Recognize that statistical inference involves drawing conclusions about a population based on data from a sample	Include that inference is not always appropriate - some questions can be answered informally from visualization, census data may be obtainable for answering directly, or too much data exceeds the limits of inference Small samples can still yield information pointing to further research
	C	Describe the existence and nature of databases and algorithms that leverage data for decision-making, including systemic biases	Describe the existence and nature of databases and algorithms that leverage data for decision-making; including systemic biases (in more sophisticated ways)	Exposure to the vastly different uses of HUGE datasets, especially predictive analytics, that exceed the objectives of this course Note that data technologies have improved to deal with the flood of data and that databases provide one form to store tabular and non-tabular datasets

Useful Visualizations: popular media presentations that have clear descriptions of data provenance

Useful Technology:

Critical Topics/Examples/Equity Connections:

- How can we make sense of education data for Indigenous people when assessments haven't been consistent?
- How can a political poll of a small sample help predict the outcome of an election, and how could poor sampling affect its predictiveness?
- If a medical study is conducted on just one type of patient, how might this create problems for the medical care for other populations?
- Can a database behind AI have biases? If so, how might that affect the way the AI behaves?

TOPIC 2: *Using technology to investigate data through visualizations and numerical summaries

The Core column contains many very typical data descriptions that appear in typical secondary mathematics curricula. The emphasis in a college level course should be on interpretation (particularly with census data) rather than computation and on choosing the most illustrative visualization rather than depending on the default settings the technology offers. While the Stretch column pushes into more modern, non-static visualizations, Core students can also benefit from interpreting displays that have not been introduced in high school or previous courses.

Core Concepts: Features of Visualization, Summarizing Data, Correlation and Causation

Prerequisite mathematics	Time on Task	CORE LOs (15%)	STRETCH LOs	Pedagogical remarks from reviewers
	B	Identify graphical displays appropriate for understanding characteristics of a given data set	Identify graphical displays appropriate for understanding characteristics of a given data set	histogram vs. stemplot based on sample size, bar vs. pie based on # of categories, two axes or side-by-side; labeling!
Plot points on a line (using tech)	C	Display univariate data (such as histograms, boxplots, dotplots and bar charts)	Display univariate data beyond those typical in print media	Construction is less important than interpretation
Plot points, definition of slope, graph a line, evaluate an equation (using tech)	C	Display bivariate data (such as scatterplots, clustered/stacked bar charts and comparative histograms or boxplots)	Display bivariate data beyond those typical in print media	Construction is less important than interpretation

	B	Display data on three or more variables in a way that allows for multivariable comparisons (e.g., using colored/shaped points and regression lines for comparison).	Display data on three or more variables in a way that allows for multivariable comparisons (e.g., using colored/shaped points and regression lines for comparison)	Stratification by one variable, leading to comparison of univariate or bivariate displays
Approx. the area of a shaded region under a curve, extract info from tables and graphs	B	Interpret displays of data via tables and visualization.	Interpret displays of data via tables and visualization that go beyond static images	Construction is less important than interpretation "anatomy of a table/graph" (without using the word "critique" that implies fault-finding)
Use tech!	C	Calculate numerical summaries of data (center, variability, and relative location) for a given data set	Calculate numerical summaries of data (center, variability and relative location) for a given data set,	Focus more on the meaning/interpretation of variability, situations where expect more/less variability, how variability is not impacted by sample size. Lots of student misconceptions to focus on here (e.g., variability vs. variety)
	B	Interpret numerical summaries to understand a given dataset	Interpret numerical summaries to understand a given dataset	Graphs of data showing mean vs. median and moving dots to explore resistance; apply to income inequality
	B	Identify elements of a visual or numerical summary of a data set that could lead to interpretation error	Use graphical and numerical summaries of a dataset to ethically communicate what the data is saying	Gaining an eye for potential error (deliberate or not) in presentation of data that obfuscates rather than illuminates

Extract information from tables and graphs	C	Extract meaning from data visualizations and numerical summaries in the media.	Extract meaning from data visualizations and numerical summaries in the media.	
	B	Assess the appropriateness of conclusions drawn from data visualizations.	Assess the appropriateness of conclusions drawn from data visualizations.	

Useful Visualizations: What's Going on in this Graph? (Link in references)

Useful Technology: spreadsheet, web apps, professional (student version) statistical package

Critical Topics/Examples/Equity Connections:

- How might the mean and median tell different stories about how well a city's school system is serving its students?
- Based on the American Community Survey, how well has household income on Indigenous tribal lands kept pace with national trends?
- Do broken-window policing policies reduce violent crime?

TOPIC 3: Handling data and understanding data distributions

This topic ties to the two previous ones as well as being the underpinnings of the statistical inference to come. Additional understanding about how raw data needs careful inspection *before* going any further bolsters confidence that statistical processes are not tainted by errors or missing values in the data. Randomness is key to understanding distributions of collected sample data and theoretical/simulated probability distributions.

Core Concepts: Data Types, Data Distributions, Features of Visualization, Variation, Dependent/Independent

Prerequisite mathematics	Time on Task	CORE Los (10%)	STRETCH LOs	Pedagogical remarks from reviewers
	C	Distinguish between raw and clean data	Distinguish between raw and clean data	Look at the collected data to identify inappropriate or missing data; knowing when given data has been cleaned
	C Spiral	Describe ethical methods for cleaning data	Apply ethical methods for cleaning data	Justify any removal of data (outliers, missing single item per observational unit, etc.) with rigorous investigation; cleaning does not mean throwing away data that doesn't support a favored interpretation

	C Spiral	Describe how randomness relates to the variability and distribution of outcomes	Describe how randomness relates to the variability and distribution of outcomes	Make sure to discuss probability distributions side by side with data distributions. Ideas of expected value and variable should reinforce each other and should not be compartmentalized. I do like the idea of thinking about the behavior of a probability distribution in context (e.g., behavior of different games with the same expected value).
	B Spiral	Use probability distributions to describe the long-run behavior of a random variable and identify likely and unlikely outcomes	Use probability distributions to describe the long-run behavior of a random variable and identify likely and unlikely outcomes	
	C	*Recognize common theoretical probability distributions (uniform, binomial, normal) by their graphs and typical numerical measures	*Recognize common theoretical probability distributions (uniform, binomial, normal) by their graphs and typical numerical measures	
		Determine the theoretical distribution that best describes a given dataset by looking at graphs, numerical measures or test results such as chi-squared.	Determine the theoretical distribution that best describes a given dataset by looking at graphs, numerical measures or test results such as chi-squared.	

Signed arithmetic	C	Use the Empirical Rule to make statements about a distribution, when appropriate (by comparison of data to theoretical normal distribution)	Use the Empirical Rule to make statements about a distribution, when appropriate (by comparison of data to theoretical normal distribution)	
	B	Describe how statistical models can be used to assess the strength of an association between two variables (quantitative or categorical)	Describe how statistical models can be used to assess the strength of an association between two variables (quantitative or categorical)	First make sure they understand the idea of a statistical model; make sure there are lots of comparisons between models and data
	C	Distinguish the independent and dependent(s) variables in bivariate data	Identify the independent and dependent(s) variables in bivariate data	
	C Spiral	Recognize that bivariate models such as regression are building blocks for more complicated multivariable models	Explain that bivariate models such as regression are building blocks for more complicated multivariable models	Focus more on the bigger idea of using models to predict and explain
	C	Identify any additional variables that may affect the relationship between two numerical or two categorical variables	Identify any additional variables that may affect the relationship between two numerical or two categorical variables	
	B	Use examples of multivariable relationships to identify limitations on two-way tables or simple regression in telling a full or accurate data story	Use examples of multivariable relationships to identify limitations on two-way tables or simple regression in telling a full or accurate data story	Focus on what it means to be confounding; how the "new" variable has to be related to both the response and explanatory variable

	C	*Use graphical displays and numerical measures to illustrate and describe distributions of data (shape, center, variability, and unusual observations)	*Use graphical displays and numerical measures to illustrate and describe distributions of data (shape, center, variability. and unusual observations)	Focus on comparing distributions.
	C	Use sample data and an understanding of sampling distributions to reason informally about population parameters	Use sample data and an understanding of sampling distributions to reason informally about population parameters	Visual inference
	B	Describe the roles of sample size, variability in the statistics and distributional shapes of statistical models	Explain the roles of sample size, variability in the statistics and distributional shapes of statistical models	

Useful Visualizations: theoretical probability distributions, building simulated distributions

Useful Technology: spreadsheet, web apps, professional (student version) statistical package

Critical Topics/Examples/Equity Connections:

- How well do race, family income, GPA, and standardized test scores predict admission to colleges and universities?
- Does exposure to lead-based paint affect child development?
- Does smoking cause lung disease?

TOPIC 4: Elements of statistical studies and their design

However well designed or implemented, every statistical study has its limitations. Even a well-done study can shine only finite light on our uncertain world. The purpose of understanding the design of a study is to illustrate the intricacies of data collection and analysis in pursuit of unbiased information to support decisions. Avoid the statistical paralysis that can be the result of focusing on perfection of every aspect of a study!

Core Concepts: Operationalizing Our Question, Data Types, Experiment Design, Sampling, Variation

Prerequisite mathematics	Time on Task	CORE LOs (10%)	STRETCH LOs	Pedagogical remarks from reviewers
	C Spiral	Recognize when given data is appropriate for a research objective	Collect data based on a research objective	
	B	Distinguish between an observational study and a randomized experiment, including their effects on conclusions drawn	Distinguish between an observational study and a randomized experiment, including their effects on conclusions drawn	
	Spiral	Identify an appropriate population to which a study's results can be generalized	Identify an appropriate population to which a study's results can be generalized	
	C	Identify subjects, cases, and experimental units in a statistical study	Identify subjects, cases, and experimental units in a statistical study	

	D1	Recognize the intricacies of designing an unbiased survey	Explain how survey design may introduce bias into data collection	*Many* survey tools are available that allow anyone to create a survey (and collect data) but what makes a survey "good" and "reliable?"
frequency, discrete, continuous	C	Identify a statistical study's quantitative or categorical variables, including units of measure or the categories	Identify a statistical study's quantitative or categorical variables, including units of measure or the categories	
	B	Identify the possible sources of variability in the design of a statistical study	Identify the possible sources of variability in the design of a statistical study	
	Spiral	Identify the limitations of a study design	Explain the limitations of the study design	

Useful Visualizations:

Useful Technology: free online survey platform where examples of well-designed and poorly designed surveys can be found/created

Critical Topics/Examples/Equity Connections:

- Are SAT prep courses effective?
- How could you design a study and/or survey to determine how various household factors affect academic performance?
- For many studies, American Indian/Alaskan Native are categorical variables. How might we make identity a continuous variable?

TOPIC 5: Sampling and Experimentation

A closer look at *how* to collect data that fits the study design. Note that a Stretch LO above says to collect data, which requires some of these LOs as pre- or corequisite learning. These concepts will also flow into the analysis work described in the next section.

Core Concepts: Sampling, Experiment Design

Prerequisite mathematics	Time on Task	CORE LOs (10%)	STRETCH LOs	Pedagogical remarks from reviewers
	C Spiral	Describe the role of random selection and random assignment in the design of statistical studies	Explain the role of random selection and random assignment in the design of statistical studies	Show how balance and representativeness are created: random selection of observational units, random assignment of treatments to observational units, random permutation of observed data (for a permutation test), random samples from the observed sample (for a bootstrap), and (hypothetical) samples from the underlying population. **more below
	C	Define simple random sampling as a process where every item in the population has an equal chance of being included in the sample, not merely a haphazard selection	Explain that simple random sampling means that every item in the population has an equal chance of being included in the sample, not merely a haphazard selection	More about why that's important and how it allows for generalization to the population. Random ≠ haphazard or unexpected

	B	Recognize the existence of sample-to-sample variability	Demonstrate an understanding of sample-to-sample variability	
	C	Distinguish between probabilistic (e.g., simple random, stratified) and non-probabilistic (convenience, intentionally biased) sampling techniques	Distinguish between probabilistic (e.g., simple random, stratified) and non-probabilistic (convenience, intentionally biased) sampling techniques	Emphasize a broader understanding of the benefits of probability-based sampling methods vs. personal subjectivity
	B	Identify when a sampling technique supports the generalization of survey and experimental results to the population from which the sample was taken	Identify when a sampling technique supports the generalization of survey and experimental results to the population from which the sample was taken	Watch out for teaching paralysis here, since almost no surveys or experiments can be fully generalized in this way. But we still find it useful to undertake surveys and experiments. ***below
	C	Identify sources of sampling bias	Identify sources of sampling bias	Incomplete sample frame, non-sampling variability, non-response, selection bias - equity issues
	C	Describe the importance of random assignment in comparative experiments	Explain the importance of random assignment in comparative experiments	Exposure to ideas of blocking, placebo, and double-blind
	C	Recognize the importance of ethical data collection, including human subjects review and informed consent	Apply proper data collection principles, including human subjects review and informed consent	
	D2	Recognize challenges to data privacy and security	Recognize challenges to data privacy and security	

**Care is needed here. I would suggest the Great Minds (Eureka Math) Algebra II course as a source for learning outcomes here (since this material is a core part of their curriculum and the CCSM). See De Veaux, Velleman, and Bock's 5th edition of "Intro Stats" for a careful approach to introduce all of these (as well as multivariate thinking in each of the first descriptive statistics chapters.

***As an example, I quoted a study that sampled Texas nutritionists regarding ice cream and bowl size. I asked students if they thought this would generalize to college educated people in Massachusetts. They uniformly said no since the sample was clearly not of the same population. But the reality is that we do generalize from whatever the sample frame is. There are quality issues (e.g., internet surveys, nonresponse) but we can't paralyze!

Useful Visualizations: illustrate sample-to-sample variability

Useful Technology: Random number generator

Critical Topics/Examples/Equity Connections:
- What are barriers to collecting a bias-free sample that is representative of an entire city's population?
- How might a particular newspaper create a poll that seems random, but has biases that are not broadly generalizable?
- How could a study that employs Indigenous Data Collection (collecting data according to tribal history or connections) do a better job of representing Indigenous cultural constructs than collecting data according to government-imposed boundaries?

TOPIC 6: Basic methods of statistical inference in a variety of contexts

The methods in this section are meant to apply to inference from either simulation or theoretical distributions. *Doing* both in the same semester is a challenge to time and cognitively difficult for students. However, there should be some introduction to the other possibility for the sake of future exposure in literature. That may include a conversation of how analysis may yield non-identical results from identical data.

Core Concepts: Law of Large Numbers, Inference, Operationalizing Our Question, Variation, Data Distribution, Experiment Design, Dependent/Independent Variables

Prerequisite mathematics	Time on Task	CORE LOs (20%)	STRETCH LOs	Pedagogical remarks from reviewers
	A	Identify appropriate inferential procedures based on the objective of an investigation and the way in which the data are collected	Identify appropriate inferential procedures based on the objective of an investigation and the way in which the data are collected	
	B	When given a large dataset with several variables (such as an observational study), describe the aspects of the data or study design that may have confounders that can lead to invalid conclusions	When given a large dataset with several variables (such as an observational study), describe the aspects of the data or study design that may have confounders that can lead to invalid conclusions	The notes for the first LO (randomness) in the Topic 6 apply here too. Confounding can lead to no observed association when there really is one just as it can lead to a Type I error
	C	Recognize the risks of drawing conclusions based on sample data	Describe risks of drawing conclusions based on sample data	

	C Spiral	Apply appropriate inference techniques for observational and experimental studies	Distinguish appropriate inference for observational and experimental studies	
	C	Describe the impact of confounding variables	Explain the impact of confounding variables and how to control (e.g., stratification or multiple regression) for them when possible	
	A	Select appropriate methods for common one- and two-sample analysis based on the objective of an investigation and the way in which the data are collected	Select appropriate methods for common one- and two-sample analysis based on the objective of an investigation and the way in which the data are collected	
Use tech!	A	Construct confidence intervals to estimate population parameters (e.g., mean, proportion) or differences between two	Construct confidence intervals to estimate population parameters (e.g., mean, proportion, variance) or differences between two	Focus on estimate +- margin of error
	A	Interpret confidence intervals in context	Interpret confidence intervals in context	
Use tech!	A	Conduct a hypothesis test about population parameters (e.g., mean, proportion) when given a research question and a clean set of data that satisfies the conditions of the test	Conduct a hypothesis test about population parameters (e.g., mean, proportion, variance) when given a research question and a clean set of data (after verifying that conditions are met)	Never do a test without a visualization of the p-value

	A	Communicate p-values as likelihood of the sample outcome (or something more extreme) under the condition that the null hypothesis is true	Communicate p-values as likelihood of the sample outcome (or something more extreme) under the condition that the null hypothesis is true	Make sure this ties to conditional probabilities.
	A	Conduct a hypothesis test about bivariate data (slope or correlation) when given a clean set of data that satisfies the conditions of the test	Conduct a hypothesis test about bivariate data (slope or correlation) when given a clean set of data (after verifying that conditions are met)	Emphasize that the strength of evidence (p-value) is not the same as the strength of the correlation
	B	Determine the validity and utility of SLR between two numerical variables through graphing and R2	Determine the validity and utility of SLR between two numerical variables through graphing and R2	
Evaluate a function	B	Use the SLR line to make predictions	Use the SLR line to make predictions	Core: When there is a single dichotomous predictor and a single continuous predictor, stratifying by the dichotomous variable and comparing two SLR models is meaningful Stretch: introduction of MLR
	C Spiral	Recognize the limitations of inferential procedures	Describe the limits of validity for inferential procedures	There are many situations where a non-random sample of found data can be used to discern useful associations
	B	Recognize that a small p-value is not always the only consideration when drawing a conclusion	Demonstrate understanding of p-value by performing reality checks before drawing a conclusion	
	C	Identify limits of extrapolation (domain of predictor variable) in regression models	Describe the risks involved with extrapolation in regression models	Explore resistance

	C	Describe potential Type I and II errors in a given setting	Describe potential Type I and II errors in a given setting and be able to describe their consequences and compare the severity	
	D1	Define the power of a statistical test by its relationship to potential errors in inference	Explain the importance of considering the power of a statistical test	Lots of graphs of null/alt distributions Stretch might also include calculation and application in different contexts.
	B Spiral	Interpret output from a variety of statistical software to support conclusions and answer research questions	Interpret output from a variety of statistical software to support conclusions and answer research questions	But not using tables (instead some form of approximation like the empirical rule). Interpreting output is much more important

Useful visualizations: theoretical or simulated null distribution

Useful Technology: web app, cloud software, spreadsheets, professional statistical packages

Critical Topics/Examples/Equity Connections:

- Are women paid fairly?
- Is the water in Flint, Michigan less healthy than water in other cities?
- How have studies in which one race is compared to another (e.g., with educational outcomes or crime data) been used to exacerbate inequities, and what other confounding factors might have been at play?

TOPIC 7: Communicating conclusions from inferential procedures

Understanding of inference includes communicating a conclusion from methods applied in pursuit of a research objective. Such communication includes more than just numbers, graphs, and statistical jargon. Formality and precision depend on the audience for statistical communication of results.

Core Concepts: Operationalizing Our Question, Inference, Features of Visualizations

Prerequisite mathematics	Time on Task	CORE LOs (10%)	STRETCH LOs	Pedagogical remarks from reviewers
	C Spiral	Clearly communicate results, including software outputs, as well as scope of conclusions and limitations of the study design	Clearly communicate results, including software outputs, as well as scope of conclusions and limitations of the study design	It's important that students: can interpret p-values, know to report the p-value (and not just say $p < 0.05$) and report an interval estimate with their p-value
	C	Explain how "not rejecting" the null hypothesis is different from "accepting" the null hypothesis	Clearly communicate that rejecting a null hypothesis implies strong support for the alternative hypothesis but that failing to reject the null hypothesis does not imply strong support for the null hypothesis.	
	C	Describe what it means to say "statistically significantly different" in the context of a hypothesis test about a difference in population means or proportions	Explain the meaning of "statistically significantly different" in the context of a hypothesis test about a difference in population means or proportions	

	C	Explain the difference between statistical significance and practical importance	Explain the difference between statistical significance and practical importance and describe the role of sample size as it relates to determining statistical significance	
	D2	Describe the use of an effect size in drawing a conclusion	Use effect size to distinguish the practicality of a result	Core: exposure to reported effect sizes Stretch: may or may not calculate
	C	Distinguish between ethical and suspicious communication of data-based conclusions	Demonstrate ethical communication of data-based conclusions	Question how the data was collected, from whom, and how they were used

Useful visualizations:

Useful Technology: office productivity software that will allow a mix of text, images, tables, and technical notation; the same tech that produced what is being communicated.

Critical Topics/Examples/Equity Connections:

- How much student loan debt do people have? Look at race, age, and educational attainment.

TOPIC 8: Relationships involving causality and/or correlation

Casual conversation imprecisely uses language that is used precisely in the context of data and its analysis. These LOs aim to move students to understand causality, association, and correlation as relationships among data that is being analyzed. Even novice users of statistics can work beyond bivariate relationships and visualize complex connections.

Core Concepts: Correlation and Causation, Features of Visualization

Prerequisite mathematics	Time on Task	CORE LOs (10%)	STRETCH LOs	Pedagogical remarks from reviewers
	C	Distinguish between association/correlation and causal relationships	Explain how association/correlation differs from causation	
	C	Use stratification to make sense of relationships between three variables that may involve confounding	Use tools, processes, or representations such as causal diagrams to make sense of causal relationships	Does this involve some examples where there are multivariate relationships that can be disentangled in straightforward ways (e.g., by stratification)? The revised GAISE College report discusses an example from Whickham, UK, where age is a confounder of the association between smoking and death. See Kari Lock Morgan's 2018 presentation at http://www.personal.psu.edu/klm47/presentations.htm It might be helpful to stick to three dimensional examples here (e.g., X = coffee drinking, Y = cancer, Z = smoking) where X and Z are associated, and Z operates causally on Y.

	C	Recognize studies that support a causal conclusion	Determine whether or not a situation involving correlation also implies causality	Traditionally our courses have focused almost exclusively on univariate and bivariate inference, yet potential confounders abound. It's important that students not finish their statistics course that culminates with the two-sample t-test and simple linear regression and be told that "Other factors" could be generating invalid associations (or making real associations disappear!). I worry that they would ask why they took a course if almost all of the data out there aren't from a randomized trial?
	C	Identify elements of study design that contribute to understanding causal relationships	Describe elements of study design that contribute to understanding causal relationships	data collection, experimental design
	B	Identify whether a correlation presented in a popular media source is appropriate or spurious	Identify whether a correlation presented in a popular media source is appropriate or spurious	Include examples of spurious correlations that are due to more than just chance; even appropriate correlation is not *sufficient* for a causal claim but may identify a future experiment

Useful Visualizations: causal diagrams

Useful Technology: same tech that creates bivariate visualizations

Critical Topics/Examples/Equity Connections:

- Do masks help prevent the spread of disease?
- Is one racial group better at some academic or athletic discipline than another?

TOPIC 9: *Being critical consumers (reviewers) of statistical arguments

It is not only the mechanics of descriptive and inferential statistics that support critical thinking when faced with data conclusions.

Core Concepts: Operationalizing Our Question, Inference

Prerequisite mathematics	Time on Task	CORE LOs (10%)	STRETCH LOs	Pedagogical remarks from reviewers
	C	Distinguish the ethical application of statistical methods from attempts to misinform or exploit using statistics	Distinguish the ethical application of statistical methods from attempts to misinform or exploit using statistics	
	C	Identify ethical issues associated with sound statistical practice	Identify ethical issues associated with sound statistical practice	some thoughtful discussion in https://mdsr-book.github.io/mdsr2e/ch-ethics.html as well as ASA Ethical Guidelines
	C	Identify ways that statistics may be used to mislead	Identify ways that statistics may be used to mislead	This is important but we need to balance with positive examples (lest students just feel that all they've learned is that one can lie with statistics).
	C	Question processes that might be used to justify foregone conclusions.	Question processes that might be used to justify foregone conclusions.	cherry-picking, "stacking the deck" in data collection, p-hacking, etc.

	C	Evaluate whether or not conclusions are appropriate for a given data collection method.	Evaluate whether or not conclusions are appropriate for a given data collection method.	It's important that students not be paralyzed after the course is completed. As an example, one could state that the only conclusions that can be causally made are from a randomized trial with full adherence and no dropout. But that never happens. Students need to understand that there is some nuance regarding "non-sampling error" to allow them to extract insights from data.

Useful Visualizations:

Useful Technology:

Critical Topics/Examples/Equity Connections:

- Pull contemporary uses of graphs in advertisements and the media
- Just because one can conduct statistically sound research does not mean it is ethically moral. For example: the Havasupai case with blood samples (https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5310710/).

Supporting Topic: Probabilistic reasoning in contexts of uncertainty

Probability and Statistics have been an inseparable pair for centuries. The rise of computing power began to fracture this relationship; the ubiquity of desktop software, web, and mobile apps to crunch the numbers diminished the reliance on the mathematics of probability for *doing* statistics. Probability is sometimes still taught by statistics instructors because there is great value in understanding all varieties of proportional reasoning and there is nowhere else to house these LOs in a typical undergraduate general education curriculum.

Prerequisite mathematics	Time on Task	CORE LO's	STRETCH LO's	Pedagogical remarks from reviewers
	D2	Distinguish situations involving uncertainty as possible, plausible, probable, or preferred	Analyze situations to distinguish between possible, plausible, probable, and preferred outcomes to make sense of situations involving uncertainty	Both logically (there is a zero chance of rolling a 0 or 7 on a six-sided die) and numerically (there is equal chance of rolling any number 1-6) Include examples that are not equally likely.
fraction/ decimal arithmetic	D2	Define probability as the likelihood of an event that must be quantified between 0-1	Define probability as the likelihood of an event that must be quantified between 0-1	
Venn diagrams, set notation	D2	Define independence, complement and mutually exclusive events	Define independence, complement and mutually exclusive events	

find the complement of a set, find union & intersection of two sets	D1	Calculate probabilities of events, including unions, intersections, and complements	Calculate probabilities of events, including unions, intersections, and complements	Mathematical calculation as well as simulation and hypothetical 1000 tables knowing the probability of an AND event must be smaller than the individual probabilities.
	D2	Distinguish between independent and dependent events	Explain the difference between independent and dependent events	
	D2	Distinguish between an unconditional probability and a conditional probability	Explain the difference between an unconditional probability and a conditional probability	
fraction/ decimal arithmetic	D1	Calculate conditional probabilities from given information, including reversed conditioning	Calculate conditional probabilities from given information, including reversed conditioning	Reversed conditioning: P(A|B) and P(B|A), positive test disease and disease positive test
	D1	Interpret probabilities in contexts from real world scenarios	Interpret probabilities in contexts from (more nuanced) real world scenarios	Long run relative frequency converging
	D1	Use probabilities of random events to predict future outcomes	Apply the independence of random events to predict the probability of a future outcome, when given a theoretical or long-run probability, as well as information about recent outcomes	
	D2	Define the risk (expected return) of an event as the product of its probability times the cost of the event occurring	Explain the risk (expected return) of an event as the product of its probability times the cost of the event occurring	

	D1	Recognize the difference between Bayesian and Frequentist reasoning	Describe the difference between Bayesian and Frequentist reasoning	Avoid the symbolic representation and stick to tables and trees; use "conditional probability." There's lots of indication that teaching this as a "hypothetical cohort" or as a tree diagram makes the concept and calculation much more straightforward than the formula for Bayes rule.
	D2	Identify at least one Bayesian and one Frequentist from among a variety of scenarios	Distinguish between Bayesian and Frequentist reasoning in a variety of real-world scenarios	
	D1	N/A	Use Bayesian reasoning to calculate the likelihood of an event	

Useful Visualizations: *Build* distributions from data, slowly starting with a physical experiment then through simulation.

Useful Technology: A simple calculator will suffice in finding proportions/percents. A spreadsheet may be helpful to record outcomes from random events (rolling a die, drawing from a hat, flipping coins/cards, etc.) before switching to a simulation. Web apps abound for building visualizations of probability distributions from either a record of outcomes or through simulation.

Critical Topics/Examples/Equity Connections:

- Does it make sense to play the lottery?
- Why does the house always win, and how can Indigenous tribes rely on gambling revenue from casinos?
- If a woman has given birth to three boys, is she more likely to have a girl next?
- With a fecal home analysis kit, if you test positive for colon cancer, what are the chances you have colon cancer?
- If a witness says they saw the defendant leave the scene of the crime, how should the jury weigh that evidence?

Suggested Resource Material

American Statistical Association

- Ethical Guidelines for Statistical Practice - https://www.amstat.org/your-career/ethical-guidelines-for-statistical-practice
- Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report - https://www.amstat.org/docs/default-source/amstat-documents/gaisecollege_full.pdf
- Resources for Undergraduate Educators - https://www.amstat.org/education/undergraduate-educators
- What's Going on in this Graph? - https://www.amstat.org/whats-going-on-in-this-graph

Mathematical Association of America's Special Interest Group (SIGMAA) on Statistics and Data Science Education - http://sigmaa.maa.org/stat-ed/

American Mathematical Association of Two-Year Colleges also has a Statistics Committee - https://my.amatyc.org/statisticscommittee/home

Dana Center Mathematics Pathways (DCMP) - https://www.dcmathpathways.org/resources/call-action-expand-access-statistics

ESTEEM: Enhancing Statistics Teacher Education with E-Modules - https://www.fi.ncsu.edu/projects/esteem/