Students can solve problems they don’t understand

Write a story problem for

\[ 6 \times 3 = \underline{\quad} \]
Students can solve problems they don’t understand

Write a story problem for

\[ 6 \times 3 = \] __________

Most common answer type in 4th and 5th grades: *There were six ducks swimming in a pond. Then a while later three more ducks come so how many are there? Six times three is eighteen. That’s the answer.*

Grade 4 37%
Grade 5 44%
There are 26 sheep and 10 goats on a ship. How old is the captain?
There are 26 sheep and 10 goats on a ship. How old is the captain? 36
Rote learning is the result of most education. Instead, teach street-fighting reasoning

1. Rote learning and its consequence

2. Street-fighting tools
   a. lumping
   b. comparing
An army bus holds 36 soldiers. If 1128 soldiers are being bused to their training site, how many buses are needed?
Students divide without understanding

An army bus holds 36 soldiers. If 1128 soldiers are being bused to their training site, how many buses are needed?

All students divided 1128/36 correctly

- 30% incorrect division
- 70% divided 1128/36 correctly

- 18% 32
- 23% 31
- 29% 31 R 12
An army bus holds 36 soldiers. If 1128 soldiers are being bused to their training site, how many buses are needed?

<table>
<thead>
<tr>
<th></th>
<th>calculator</th>
<th>paper/pencil</th>
</tr>
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<tbody>
<tr>
<td>right</td>
<td>18 (7.2%)</td>
<td>59 (23.6%)</td>
</tr>
<tr>
<td>wrong</td>
<td>232</td>
<td>191</td>
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P(calculator helped or did no harm| data) \approx 10^{-7}. 

Using a calculator harmed students’ performance
Students need to turn on their minds, not their calculator

Estimate $3.04 \times 5.3$

1.6
16
160
1600
No answer
Students need to turn on their minds, not their calculator

Estimate $3.04 \times 5.3$

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<table>
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</tr>
<tr>
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<td>21</td>
<td></td>
</tr>
<tr>
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Rote learning happens at all educational levels

Is $\ln 7!$ greater than or less than $\int_{1}^{7} \ln k \, dk$?
Rote learning happens at all educational levels

Is $\ln 7!$ greater than or less than $\int_{1}^{7} \ln k \, dk$?
Rote learning happens at all educational levels

Students reasoned using only numerical calculation:

$$\int_{1}^{7} \ln k \, dk = k \ln k - k \bigg|_{1}^{7} \approx 7.62.$$  

$$\ln 7! = \sum_{1}^{7} \ln k \approx 8.52.$$  

$$\sum \frac{8.52}{7.62} > \int$$
Rote learning combines the worst of human and computer thinking

<table>
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<tr>
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<th>computer chess</th>
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<tbody>
<tr>
<td>calculation</td>
<td>1 position/second</td>
<td>$10^8$ positions/second</td>
</tr>
<tr>
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<td>fantastic</td>
<td>minimal</td>
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Rote learning combines the worst of human and computer thinking

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Rote learning is the result of most education. Instead, teach street-fighting reasoning

1. Rote learning and its consequence

2. Street-fighting tools
   a. lumping
   b. comparing
Street fighting is the pragmatic opposite of rigor

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MIT Press, 2010
Street fighting is the pragmatic opposite of rigor
Street fighting is the pragmatic opposite of rigor mortis
Every number is of the form:

\[
\begin{pmatrix}
\text{one} \\
\text{or} \\
\text{few}
\end{pmatrix} \times 10^n,
\]

where

\[\text{few}^2 = 10.\]
Street-fighting tool 1: Simplify using lumping

How many seconds in a year?

\[
\frac{365 \text{ days}}{\text{year}} \times \frac{24 \text{ hours}}{\text{day}} \times \frac{3600 \text{ seconds}}{\text{hour}}
\]
How many seconds in a year?

\[
\frac{\text{few} \times 10^2 \text{ days}}{\text{year}} \times \frac{24 \text{ hours}}{\text{day}} \times \frac{3600 \text{ seconds}}{\text{hour}}
\]
Street-fighting tool 1: Simplify using lumping

How many seconds in a year?

\[
\frac{\text{few} \times 10^2 \text{ days}}{\text{year}} \times \frac{\text{few} \times 10^1 \text{ hours}}{\text{day}} \times \frac{3600 \text{ seconds}}{\text{hour}}
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Street-fighting tool 1: Simplify using lumping

How many seconds in a year?

\[
\frac{\text{few} \times 10^2 \text{ days}}{\text{year}} \times \frac{\text{few} \times 10^1 \text{ hours}}{\text{day}} \times \frac{\text{few} \times 10^3 \text{ seconds}}{\text{hour}}
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\]

\[
\sim \text{few} \times 10^7 \text{ seconds/} \text{year}
\]
Lumping also works on graphs
Pictures explain most of Stirling’s formula for $n!$

$$\ln n! \approx \int_1^n \ln k \, dk = n \ln n - n + 1;$$

$$n! \approx e \times \frac{n^n}{e^n}.$$
The protrusions are the underestimate
Each protrusion is almost a triangle
Doubling each triangle makes them easier to add
The doubled triangles stack nicely

Sum of doubled triangles = $\ln n$
The integral along with the triangles explain most pieces of Stirling’s formula for $n!$

$$\ln n! = \sum_{1}^{n} \ln k$$

$$\approx n \ln n - n + 1 + \frac{1}{2} \ln n$$

$$n! \approx e \times \frac{n^n}{e^n} \times \sqrt{n}$$

should be $\sqrt{2\pi}$
Rote learning is the result of most education. Instead, teach street-fighting reasoning

1. Rote learning and its consequence

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   a. lumping
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Hard problems demand more street-fighting methods

What is the fuel efficiency of a 747?
The rote method is hopelessly difficult

Equations of fluid mechanics

\[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} \]

\[ \nabla \cdot \mathbf{v} = 0 \]

where

\( \rho \) = air density
\( p \) = pressure
\( \mathbf{v} \) = velocity
\( \nu \) = (kinematic) viscosity
\( t \) = time
What is the approximate ratio of the fall speeds $v_{\text{big}}/v_{\text{small}}$?

a. 2 : 1

b. 1 : 1

c. 1 : 2
What is the approximate ratio of the fall speeds $v_{\text{big}}/v_{\text{small}}$?

a. $2:1$

b. $1:1$ Drag force is proportional to area!

c. $1:2$
We need a short interlude with a symmetry principle

\[
\text{drag force} \sim \text{area} \times \text{density \? speed \? viscosity ?} \\
\frac{\text{kilograms} \times \text{meters}}{\text{second}^2} \times \frac{\text{meters}^2}{\text{meters}^2} \times \frac{\text{kilograms}}{\text{meter} \times \text{second}^2}
\]
We need a short interlude with a symmetry principle

\[
\text{drag force } \sim \text{area} \times \text{density} \times \text{speed}\text{? viscosity}?
\]

\[
\underbrace{\text{kilograms} \times \text{meters}}_{\text{second}^2} \times \underbrace{\text{meters}^2}_{\text{meters}^2} \times \underbrace{\text{kilograms}}_{\text{meter}^3} \times \underbrace{\text{speed? viscosity}}_{\text{second}^2}
\]
We need a short interlude with a symmetry principle

\[ \text{drag force} \sim \text{area} \times \frac{\text{density}}{\text{speed}^2} \]

\[ \frac{\text{kilograms} \times \text{meters}}{\text{second}^2} \times \frac{\text{meters}^2}{\frac{\text{kilograms}}{\text{meter}^3}} \times \frac{\text{meters}^2}{\text{second}^2} \]
Fuel consumption is proportional to the drag force, and

\[ \text{drag force} \sim \text{area} \times \text{density} \times \text{speed}^2. \]

The ratio of plane-to-car fuel consumptions is therefore

\[
\frac{\text{plane consumption}}{\text{car consumption}} \sim \frac{\text{area}_{\text{plane}}}{\text{area}_{\text{car}}} \times \frac{\text{density}_{\text{plane}}}{\text{density}_{\text{car}}} \times \frac{\text{speed}_{\text{plane}}^2}{\text{speed}_{\text{car}}^2}
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\]
Fuel consumption is proportional to the drag force, and 
drag force $\sim$ area $\times$ density $\times$ speed$^2$.

The ratio of plane-to-car fuel consumptions is therefore

$$\frac{\text{plane consumption}}{\text{car consumption}} \sim \frac{\text{area}_{\text{plane}}}{\text{area}_{\text{car}}} \times \frac{\text{density}_{\text{plane}}}{\text{density}_{\text{car}}} \times \frac{\text{speed}^2_{\text{plane}}}{\text{speed}^2_{\text{car}}} \sim 300.$$

But 300 passengers on a plane flight; only 1 passenger in a car.

Planes and cars are equally fuel efficient!
The connection between falling cones and flying planes helps us estimate the cost of a plane ticket.

A New York–Stockholm roundtrip is roughly 12,000 km. 

\[
12,000 \text{ km} \times \frac{8 \text{ litres}}{100 \text{ km}} \times \frac{0.5 \text{ euros}}{1 \text{ litre}} \approx 500 \text{ euros.}
\]
Connections are more important than facts alone

big cluster = 22%
p_{bond} = 0.40
Connections are more important than facts alone

big cluster = 22%  
\(p_{\text{bond}} = 0.40\)

big cluster = 68%  
\(p_{\text{bond}} = 0.50\)
Connections are more important than facts alone

- Big cluster = 22%
  \( p_{\text{bond}} = 0.40 \)

- Big cluster = 68%
  \( p_{\text{bond}} = 0.50 \)

- Big cluster = 80%
  \( p_{\text{bond}} = 0.55 \)
Connections are more important than facts alone

big cluster = 22%  
$p_{\text{bond}} = 0.40$

big cluster = 68%  
$p_{\text{bond}} = 0.50$

big cluster = 80%  
$p_{\text{bond}} = 0.55$

big cluster = 93%  
$p_{\text{bond}} = 0.60$
Rote learning is the result of most education. Instead, teach street-fighting reasoning

The goal [of teaching] should be, not to implant in the students’ mind every fact that the teacher knows now; but rather to implant a *way of thinking* that enables the student, in the future, to learn in one year what the teacher learned in two years.

**Only in that way can we continue to advance from one generation to the next.**

—*Edwin T. Jaynes (1922–1998)*