Mathematics for the Modern World -

Sample Threads (core concepts, content, learning progressions)

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Table of Contents

Sample Threads

This document is designed to illustrate how threads connect standards across the curriculum. While not every standard is *directly* linked to the other standards of that thread, they are all connected because the teachers would point out the thread running through them all. This applies to all types of threads, including Core Concepts (Discipline and Branch level), Content, and Learning Progressions.

Core Concept Example

Variables

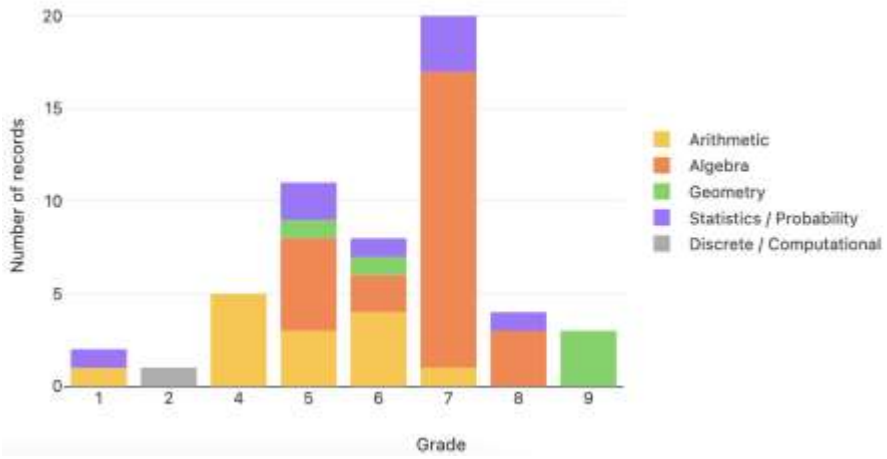
A variable is a mathematical representation of an unknown. Variables are used to express relationships generally as well as to represent desired solutions during mathematical computation. A variable may represent a potentially infinite set of values.[1]

This graph shows where the BCC ***Variables*** shows up across the curriculum.

The large block of Algebra in 7th grade is where there is a big emphasis on functions and modeling.

Before that, as early as first grade, approaches to problems at more foundational steps can set students up for a deeper understanding of variables.

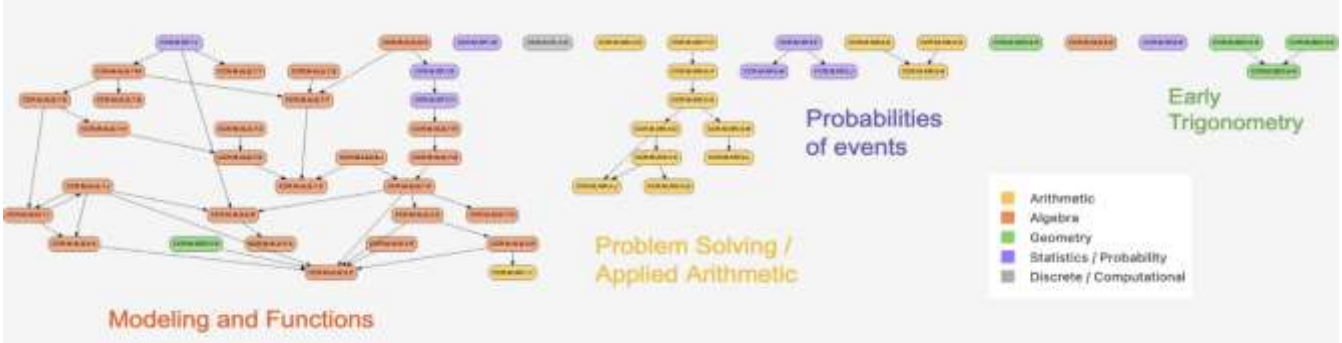
For example:



[1] For more on Core Concepts, see CCR's Knowledge Paper here.

Code	Name	Given	Asked	Students can
CCR.M.ARI.1.I	Connection between addition and subtraction	a representation of an addition problem including missing addend equations	to express the problem symbolically and visually and solve it	recognize and explain the connection between addition and subtraction and apply this relationship to solve numerical and contextual problems.

There are general patterns of when variables come up across the curriculum: modeling and functions, problem solving / applied arithmetic, probabilities of events, and early trigonometry:



A full list of the standards tagged with the BCC "Variables" can be found below:

Code	Name	Given	Asked	Can
CCR.M.ARI.1.I	Connection between addition and subtraction	a representation of an addition problem including missing addend equations	to express the problem symbolically and visually and solve it	recognize and explain the connection between addition and subtraction and apply this relationship to solve numerical and contextual problems.

Code	Name	Given	Asked	Can
CCR.M.SP.1.B	Intro to writing questions	a general authentic topic	to identify a question of interest based on one categorical variable and to gather data relevant to the question	formulate a question, collect answers, and record data systematically
CCR.M.DC.2.G	Unambiguous Sequence	an unambiguous sequential pattern presented visually, functionally, or cyclically	to articulate a rule to describe the pattern	index the pattern and use that to describe how to find additional terms using informal language
CCR.M.ARI.4.Q	Distributive law in multiplication	one or more examples of solutions to a multiplication problem involving the distributive law	to explain the reasoning presented in each example solution	interpret the steps of the solution shown and explain why it is (or is not) correct with reference to the distributive law
CCR.M.ARI.4.O	Fractions and decimals in context by hand	an authentic real-world problem involving simple division and multiplication of decimals and/or fractions	to solve, interpret the results, and justify their reasoning	write or verbalize an equation that represents the problem, solve by hand, and interpret the result in the context of the problem
CCR.M.ARI.4.J	Additive/Multiplicative Contexts by hand or mentally	an authentic real-world problem involving simple arithmetic computation with positive rational numbers	to find a solution by hand or using mental arithmetic	formulate the question, use efficient mental and/or written strategies to solve and interpret the results in context.
CCR.M.ARI.4.G	Applications of Arithmetic	an authentic real-world problem involving positive whole numbers, fractions, and/or decimals using the four operations, e.g., problems money, time, length, area, volume, mass/weight, items bought/sold	to solve the problem and explain your answer	express the problem mathematically, solve the problem, and interpret the solution in the context

Code	Name	Given	Asked	Can
CCR.M.ARI.4.K	Additive/Multiplicative Contexts generally	a problem scenario involving arithmetic computation of positive rational numbers	to find a solution	formulate the problem, use efficient strategies including appropriate digital technologies to solve, and interpret the results in context
CCR.M.ARI.5.M	Problems with percents by hand	an authentic real-world problem involving simple percentage calculations	to solve and explain your answer	formulate the problem, solve by hand, and interpret the result in context
CCR.M.ARI.5.L	Basic percent	an authentic real-world problem involving a fraction of a whole that is easily expressable as a percentage	to express as a percentage	express the equivalence between “parts per hundred” and "percent" using appropriate notation
CCR.M.ARI.5.N	Problems with percents, fractions, and/or decimals generally	an authentic real-world problem involving rational numbers and/or percent	to solve and explain your answer	formulate the equation, solve with digital assistance, and interpret the result in context
CCR.M.GEO.5.B	Solving for unknown angles inside of polygons	a diagram made up of polygonal regions with one or more unknown, but uniquely specified, angles	to determine the unknown angle(s) measurement(s)	use mathematical reasoning to determine the measure of unknown angles
CCR.M.ALG.5.E	Products involving zero	that the product of two expressions involving one or more unknowns is equal to zero	what must be true about the expressions	recognize that one or more expressions must be zero and draw conclusions about the values of the unknowns

Code	Name	Given	Asked	Can
CCR.M.ALG.5.C	Variables	an authentic real-world problem including a mathematical relationship that can be represented as an expression involving one or two unknowns	to express the relationship mathematically	identify and name either the one or two necessary variables in relation to the known quantities
CCR.M.ALG.5.D	Solving a linear equation	a one or two step linear equation expressed mathematically with a context and, optionally, a calculator	to solve and explain your answer	use learned arithmetic (including a calculator, optionally) to isolate the variable to solve for the unknown and interpret their result in the context
CCR.M.ALG.5.B	Inverse Relationships	a rational number	to find the number that when added or subtracted to the given number results in zero (additive inverse) or that when multiplied by the given number results in one (multiplicative inverse)	find the multiplicative or additive inverse of a given rational number and verify their answer
CCR.M.ALG.5.A	Define a sequence from a scenario	an authentic real-world problem with two numerical quantities and enough pairs of elements in a sequence to determine a rule relating the quantities	to describe a rule that will determine one quantity as a function of the other	determine and verify an informal rule that generates the next unknown value in the sequence that matches the given problem
CCR.M.SP.5.M	Independent events, uneven probabilities	two independent familiar events with uneven probabilities	to determine the probability of the sequence of events	list all the possible outcomes, and develop P(Event A and Event B) = P(Event A)*P(Event B) to compute the probabilities of each outcome; then check that probabilities add up to 1.

Code	Name	Given	Asked	Can
CCR.M.SP.5.J	Dependent events, uneven probabilities	two dependent familiar events with uneven probabilities	to determine the probability the combined event	create a visual representation, e.g. a tree, of the possible events, and use the representation to compute the appropriate probabilities i.e. P(Event A and Event B) = P(Event A)*P(Event B given Event A)
CCR.M.ARI.6.E	[LO/EXT] Laws of index notation, fractional powers	a pair of numbers a^m and a^n with m and n fractional numbers	to determine (a^m)(a^n) or (a^m)/(a^n) or (a^m)^n	explain the reasoning behind index notation laws, apply the laws, and explain why index laws don't apply when the bases don't match
CCR.M.ARI.6.H	Multistep proportional relationships and percents	a multi-step authentic real-world problem involving proportional relationships expressed as fractions and/or percents	to solve the problem and interpret in context	express proportional relationships in equations and solve; interpret the solutions in context
CCR.M.ARI.6.B	Index notation, whole number powers	a pair of expressions x^m and x^n where m and n are whole numbers	to explain how to combine them into x^p (a single base with an index)	explain the reasoning behind the index notation laws i.e., expand to show that multiplying the two expressions is equivalent to finding the sum of the indices, and dividing the expressions is equivalent to finding the difference between the indices
CCR.M.ARI.6.D	[LO/EXT] Index notation, integer powers	a pair of numbers a^m and a^n with m, n integers	to determine (a^m)(a^n) or (a^m)/(a^n) or (a^m)^n	explain the reasoning behind index notation laws and apply the laws and explain why index laws don't apply when the bases don't match

Code	Name	Given	Asked	Can
CCR.M.GEO.6.H	Areas of circles	A circle and measuring tools	to decompose the circle into approximate known shapes to find the area	apply a consistent smaller shape (e.g., using graph paper) to determine the approximate area and verify that it closely matches the area as calculated by pi*r^2. Recognize that the smaller the shape, the closer the estimate is to the true area.
CCR.M.ALG.6.B	Intro graphs of functions	a graph of a functional relationship, with or without a context	to interpret the graph	describe qualitatively the relationship between two quantities by analyzing a graph (e.g., point to where the function is increasing or decreasing, positive or negative, linear or nonlinear)
CCR.M.ALG.6.A	Matching Inputs and Outputs in Functions	an authentic real-world context involving two quantities and related representations of input and output pairs (e.g. graphs, tables, visuals etc.) of a linear or non-linear function	to write or verbalize a rule that maps input values to output values	identify and name an independent variable and a dependent variable. Then express a rule verbally or symbolically that generates each output from the corresponding input value for all terms in the given representation
CCR.M.SP.6.F	Probability of a sequence of events	a sequence of two events and information to determine the probabilities	to calculate the probability of that sequence of events	determine based on the scenario whether the events are independent, and apply P(A and B) = P(A)*P(B), or dependent and apply P(A and B) = P(B|A)*P(A)

Code	Name	Given	Asked	Can
CCR.M.ARI.7.C	Multi-step rates and other proportional relationships	an authentic real-world problem including a multistep problem involving rates, ratios as fractions, and/or percents	write or verbalize an equation that represents the problem and solve	fluently choose useful representations to express proportional relationships, solve, and interpret their solutions in context
CCR.M.ALG.7.O	Linear proportional relationships in context	an authentic real-world problem involving a proportional relationship between two quantities	to represent the proportional relationship by an equation, and apply and graph the equation	identify the constant of proportionality, write an equation representing the relationship in the form y = kx, where k is the constant of proportionality, apply the equation in context and graph the relationship
CCR.M.ALG.7.I	Using function notation	an authentic real-world problem involving a functional relationship between two quantities presented in a table, graph, or as a rule	to write an equation representing the problem using function notation	identify dependent and independent variables and write an equation to represent the function using function notation
CCR.M.ALG.7.P	Linear function representations	a linear relationship presented in a variety of ways (e.g., as a table of values, a graph, or a pair of points)	represent the relationship verbally, symbolically, and graphically, and evaluate for one or more new input values	sketch linear graphs using the coordinates of two points, write the linear function verbally and symbolically, and use the linear function to determine other values
CCR.M.ALG.7.Q	Plotting linear relationships by hand	a linear equation with small integer coefficients	to graph	choose appropriate scales for the axes, and plot the line on the coordinate plane
CCR.M.ALG.7.R	Plotting linear relationships generally	a linear equation, possibly with non-integer coefficients and graphing technology	to graph	plot linear relationships on the coordinate plane using graphing technologies

Code	Name	Given	Asked	Can
CCR.M.ALG.7.T	[EXT] Exponential growth in the wild	familiar surroundings and experiences, including the internet	to identify realistic situations that demonstrate exponential growth pattern and identify variables, sketch a graph representing the situation	identify at least three situations from different fields, recognize the variables, and sketch the graphs of the relationships between variables
CCR.M.ALG.7.H	Explain parameters	an authentic real-world problem, and a formula or a graph	to interpret the formula or graph in terms of the problem	explain the details that correspond to parameters (e.g., rate of change, intercepts) of the model in context
CCR.M.ALG.7.D	Choose best Option Using Graphing	data from a real-world scenario and graphing software with multiple models some of which may be unfamiliar	to analyze different models and select the most appropriate representation for the situation	select appropriate parameters in a graphing program to generate the best model, and justify their choice
CCR.M.ALG.7.F	Recognizing models from patterns	a description of a pattern over time (e.g., a recursive formula, a statement that growth is constant over time, etc.)	sketch the shape of the curve to identify the appropriate model	sketch the shape of the curve and determine the type of model (linear, constant, quadratic, periodic, exponential, logistic)
CCR.M.ALG.7.M	Using models to make predictions	a rule in context or data expressed graphically, in a table, etc. that takes place over time	to predict an approximate value at a given time	create a model that reflects the given data and use it to make predictions in the form of a range

Code	Name	Given	Asked	Can
CCR.M.ALG.7.K	Sketching graphs by hand	a function represented algebraically or through an authentic real-world problem	to sketch a graph that represents the function	sketch a graph that represents that function taking into account provided parameters to a reasonable degree of accuracy in the context of the problem including selecting and noting appropriate scale and units
CCR.M.ALG.7.B	Determine a simple exponential formula	a real-life exponential growth or decay scenario descriptively or in the form of a table	to calculate value(s) that are inefficient to calculate iteratively	identify the need for, create, and manipulate an iterative formula into a closed exponential form (y=ak^x)
CCR.M.ALG.7.G	Overtaking	an authentic, real-world scenario with multiple types of growth or decay (e.g., exponential, polynomial, etc.) represented as a graph, table, or description	to identify which is growing/decaying faster, if/when one will overtake the other	with or without technology, identify which quantity is growing faster and will therefore overtake the other regardless of initial value
CCR.M.ALG.7.J	Function evaluation	a scenario and a rule expressed as a graph or in function notation and some inputs or outputs	to evaluate the function on the inputs or to determine the input that corresponds to a given output	evaluate the function to find outputs at certain inputs and find inputs that correspond with given outputs.

Code	Name	Given	Asked	Can
CCR.M.ALG.7.E	[LO] Not Enough Info and Narrowing Down	multiple data points in context where measurement error and/or quantity make them insufficient to define a growth pattern	draw in a model and explain their reasoning	recognize there is insufficient data to determine exactly one model that fits the data, draw (no need to name) two or more possible models that fit the data (linear, constant, quadratic, periodic, exponential, logistic), and state which, if any, familiar models are eliminated given the data (e.g., linear and constant)
CCR.M.SP.7.G	Correlation (r values)	realistic bivariate data from a scenario and a graphing technology to find linear regressions	to use the r value to discuss how strong the correlation is between the data	explain that r~1 is a strong direct correlation, r~-1 is a strong negative correlation, and r~0 is no correlation
CCR.M.SP.7.H	Express bivariate data	a bivariate numerical data set and graphing technology (for making scatterplots)	to describe the relationship based on the scatterplot	express the relationships between the variables graphically and verbally (e.g., linear / non-linear, unusual values, strength, direction)
CCR.M.SP.7.L	Policy Makers	an authentic real-world growth scenario (potentially with a specified delay in data availability) and one or more possible interventions represented mathematically	make a decision about how to respond	use tables, iterative and closed formulas, and/or graphs to make a sound decision and justify the decision.

Code	Name	Given	Asked	Can
CCR.M.ALG.7.C	Sketching Curves	a scenario about causality (e.g., one person with a disease infects 2 people every day)	to sketch a curve that represents the scenario	sketch a curve with accurately labeled axes and name the appropriate type of relationship (e.g., linear, exponential)
CCR.M.ALG.8.A	[EXT] Interpreting growth/decay, general	given a table of realistic data in a context	to determine the type of growth	calculate differences and determine whether growth/decay is sublinear (e.g., logarithmic), linear, or superlinear (e.g., polynomial, exponential)
CCR.M.ALG.8.J	Modeling using technology	an authentic real-world problem including data and technology	to identify a question, model the data, solve, and present your results	express, solve, and interpret solutions using technology to models using tables, graphs, equations, and descriptions
CCR.M.ALG.8.H	Curve fitting	a realistic data set and graphing technology	to find the best fitting curve	use graphing technology to find the best fitting curve, whether it's quadratic, linear, exponential, or periodic and explain why it was the best fit using the r value.
CCR.M.SP.8.B	Skew, symmetric, bimodal	a data set	to express and/or interpret the data set appropriately explaining the results	choose an appropriate visual representation of the data and interpret the results using terms including “skewed,” “symmetric,” and “bi modal”

Code	Name	Given	Asked	Can
CCR.M.GEO.9.D	Sine, Cosine	a right-angled triangle and some additional information about angle(s) and/or side length(s)	what can you determine about angles and side lengths	use the definitions of sine, cosine, and the Pythagorean Theorem to determine all side lengths and angles for the triangle, or explain why there are multiple triangles that satisfy the known information via similarity
CCR.M.GEO.9.G	Applying right-triangle trigonometry	a scenario involving one or more right triangles	to solve a problem	apply trigonometry to solve right-angled triangle problems including those involving three dimensions, such as direction and angles of elevation and depression
CCR.M.GEO.9.E	Determining sides and angles in a quadrilateral	a classification of a quadrilateral with some labeled angles and/or sides	to determine the other angles and/or sides	find all unknowns or identify those that are unknowable from the information provided

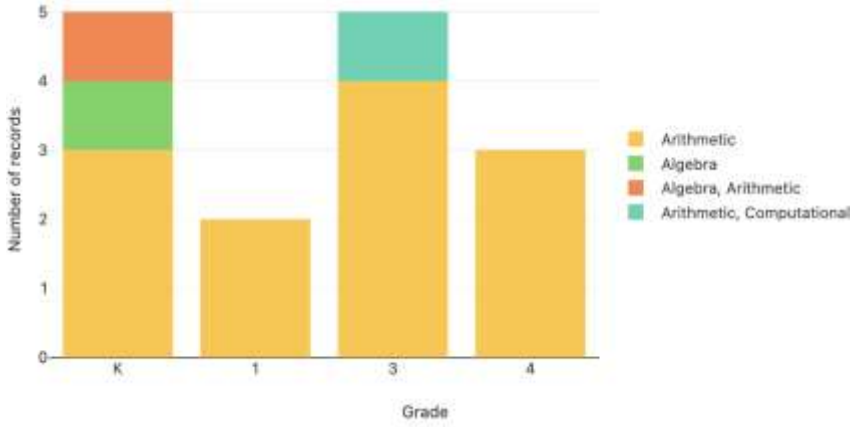
Content Thread Example

Content threads follow traditional throughlines across the curriculum. This is the place where those unfamiliar with the CCR framework should look for the terms they would be most familiar with. Here we trace the example of:

Place Value

the number system in which the position of a digit in a number determines its value

This graph shows where the Content Thread ***Place Value*** shows up across the curriculum.



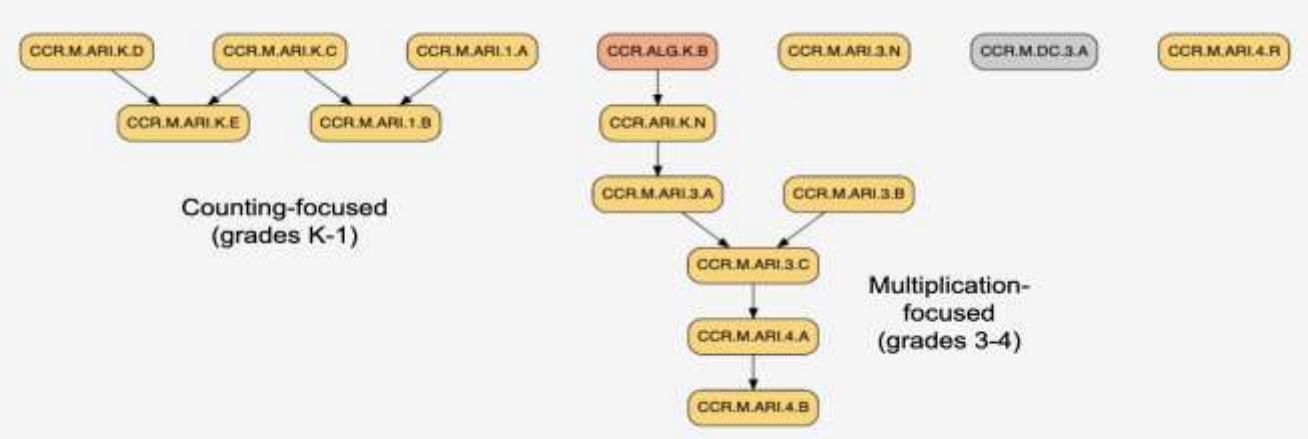
Unsurprisingly, the majority of the standards which include Place Value are in the Arithmetic branch. In many countries, this branch is even referred to as "Number."

The Algebra standard is about skip counting patterns, and several other standards overlap the Arithmetic with the Computational. In that one, students approach Place Value from an algorithmic perspective, devising an algorithm to compare whole numbers.

Many of the standards are not written to be *about* place value per se, but in mastering them, students are deeply practicing place value. For example:

Code	Name	Given	Asked	Students can
CCR.M.ARI.K.C	Locate a number on a number line up to 100	a partial number line representing 0 -100 with multiples of ten labeled and ...	to locate the number	locate the number on the given number line by finding the multiple of 10 first, and then adding the "ones"

There are general patterns of when Place Value comes up across the curriculum, namely: counting and multiplication. There are also one-off moments where it is tied back in even as the focus is on something else.



A full list of the standards tagged with the Content Thread “Place Value” can be found below:

Code	Name	Given	Asked	Can
CCR.M.ARI.K.D	Recognize, model, read and write numbers up to 100	a numerical value between 0 and 100 represented verbally, symbolically, or concretely	to represent that number alternatively verbally, symbolically, or concretely as a number of objects (different from the given)	connect number names, numerals and manipulatives including base ten blocks
CCR.M.ARI.K.C	Locate a number on a number line up to 100	a partial number line representing 0 -100 with multiples of ten labeled and appropriate markings for each whole number, and a specific number between 0 and 100	to locate the number	locate the number on the given number line by finding the multiple of 10 first, and then adding the “ones”
CCR.M.ARI.K.E	Order numbers to 100	a scenario involving numbers between 0 and 100 (verbally or symbolically)	to order the numbers	order the numbers and explain their process

Code	Name	Given	Asked	Can
CCR.ALG.K.B	Skip counting patterns	a beginning number and a difference to skip count by	to describe number patterns formed by skip-counting	generate the set of numbers by skip counting and note the repeating pattern in the ones digit
CCR.ARI.K.N	Patterns in counting by 10s	different starting numbers	to describe the number pattern formed by adding ten at a time from any starting point	describe the incrementing pattern in the tens digit (0 to 9)
CCR.M.ARI.1.A	Numbers to 1,000,000	a scenario involving a number between 0 and 1,000,000 (verbally or written)	to show that number verbally, symbolically, visually, or as a physical representation (different from the given)	connect number names, numerals and visual/physical representations, and for large numbers explain how and why to use a group-based representation.
CCR.M.ARI.1.B	Place value approximation	a whole number with or without context	to round the number to the tens, hundreds, or thousands place	recognize the idea of counting by groups of 10, 100, or 1000 and round the number up or down as needed and contextualize their result if applicable
CCR.M.ARI.3.C	Compare decimals to the hundredths	two or more decimal numbers within hundredths	to compare and/or order the numbers	order the numbers and compare the values using the language of place value to hundredths and express using the symbols <, >, =
CCR.M.ARI.3.A	Place value at 10ths and 100ths	a number in decimal format (not exceeding hundredths)	express the value using tenths and hundredths	recognize that the place value system can be extended to tenths and hundredths and correctly convert the decimal to a fraction

Code	Name	Given	Asked	Can
CCR.M.ARI.3.N	Estimating multiplication and division of large numbers	a mathematical or an authentic real-world problem involving multiplication or division of large numbers	to estimate the answer quickly using mental arithmetic	use mental strategies to estimate; for example, rounding and using a multiplication fact to arrive at a reasonable estimate (e.g., 493/5 → 500/5 = 100)
CCR.M.ARI.3.B	[EXT] Place value beyond 1000ths	a number in decimal format (exceeding hundredths)	express the value with a fraction or with the sum of multiple fractions (each of which is a power of ten)	recognize that the place value system can be extended and correctly convert the decimal to a fraction
CCR.M.DC.3.A	Algorithm for comparing whole numbers	two or more whole numbers	to explain how to compare (or order) the values	explain an algorithm using informal language to compare whole numbers using place value
CCR.M.ARI.4.B	Multiply or divide powers of 10	an authentic real-world problem involving multiplication and/or division by powers of 10	to solve and explain your answer	express, solve, and interpret the solution by hand using place value
CCR.M.ARI.4.A	Decimal place value - general	a decimal representation of a number with more than two decimal places	to interpret the value	read the decimal number and re-write it as fraction or a mixed number (e.g., 5.678 = 5 678/1000)
CCR.M.ARI.4.R	[EXT] Writing numbers in different bases	a number in a given base and a different base	to write the number in the new base	write the given number in the new base

Learning Progression Example

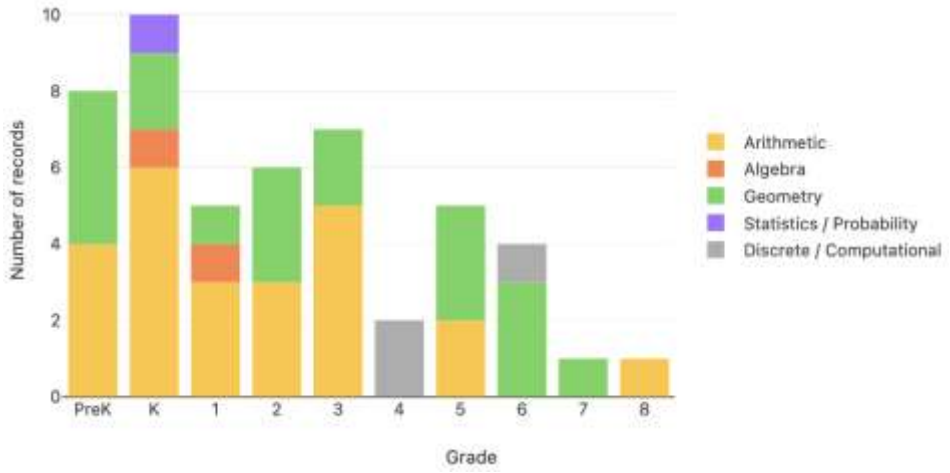
Learning progressions are threads that are traditionally thought of as following a

developmental process. They are not just a piece of content that gets explored, but they are also not Core Concepts which remain relevant across the K-12 curriculum. The following is an example of a Learning Progression: Unitizing

Unitizing

> "Unitizing is the cognitive assignment of a unit of measurement to a given quantity; it refers to the size chunk one constructs in terms of which to think about a given commodity. For example, given a case of cola, one could think of it as 24 (cans or 1-units), 2 (12-packs), or 4 (6-packs).
>
> The ability to form and operate with increasingly complex unit structures appears to be an important mechanism by which more sophisticated reasoning develops. Research in proportional reasoning, for example, indicates that one of the most salient differences between proportional reasoners and non-proportional reasoners are adept at building and using composite extensive units and that they make decisions about which unit to use when choices are available".[2]

This graph shows where the LP Unitizing shows up across the curriculum.



Geometry plays an auxiliary role across the curriculum, whereas Arithmetic is the primary branch of this LP. There are also the other three branches sprinkled in.

As a result, Unitizing is a broad, extensive Learning Progression, which covers all the branches and nearly all the grades. It is addressed both explicitly in the standards, and highlighted where it is appropriate to include in standards that are focused on other things.

[2] Lamon, S. J. (1996). The development of unitizing: Its role in children's partitioning strategies. *Journal for Research in Mathematics Education*, *27*(2), 170-193.

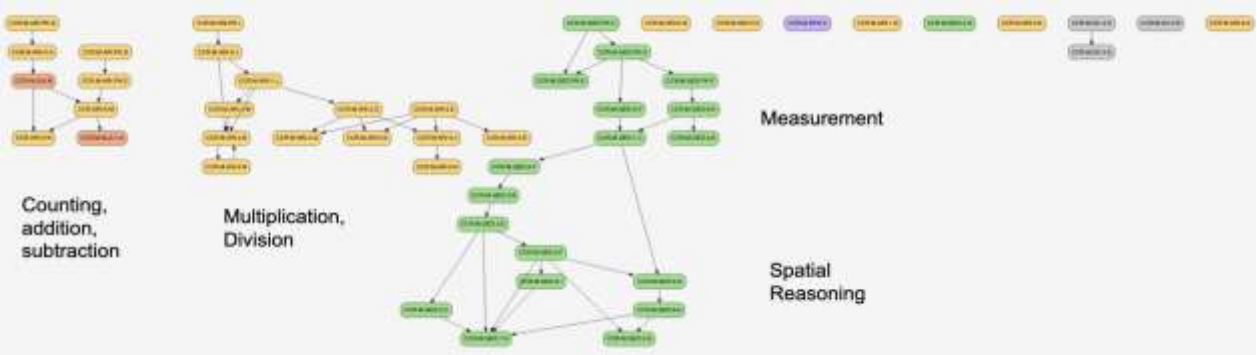
For example, this standard focuses on Unitizing:

Code	Name	Given	Asked	Students can
CCR.M.GEO.K.F	Uniform informal unit	a large object	to measure it with informal units	select an informal unit for measuring and express the answer in terms of those units (e.g. find its length using paperclips, volume using smaller containers, or weight using balance scale and oth...

Whereas this standard incorporates unitizing applied to another piece of math:

Code	Name	Given	Asked	Students can
CCR.M.ARI.2.E	Fractions as equal subdivisions of a collection	a scenario involving equal divisions of a collection	to express symbolically and visually	express symbolically and visually unit fractions including 1/2, 1/3, 1/4, 1/5 and explain how many equal portions make one collection

There are general patterns of when Unitizing comes up across the curriculum: modeling and functions, problem solving / applied arithmetic, probabilities of events, and early trigonometry:



A full list of the standards tagged with the LP “Unitizing” can be found below:

Code	Name	Given	Asked	Can
CCR.M.ARI.PK.E	Conceptual Subitizing to 7	a collection of up to 7 objects in a situation in which it is infeasible to	how many	group by 2s or 3s (into 1-3 groups)

Code	Name	Given	Asked	Can
		count (e.g., time limit)		
CCR.M.ARI.PK.G	Matching objects for subtraction, Including 0	an oral and written un-mathematized subtraction situation in a familiar context involving two sets of objects with a difference of less than 20 and various concrete materials	to identify how many more are in the larger set	solve the problem by matching up objects until one group has more than the other, or there are none left, and counting the number of objects that remain
CCR.M.ARI.PK.D	[LO] Perceptual Subitizing to 3	a collection of up to 3 objects in a situation in which it is infeasible to count (e.g., time limit)	to recognize how many	recognize and say the number of objects without counting
CCR.M.ARI.PK.I	Equal shares	a physical representation of sharing a collection or whole between a group of people	to identify whether the sharing suggestion is even	explain the concept of equal, as opposed to unequal, divisions in informal language relating to both size and quantity
CCR.M.GEO.PK.C	[LO] Language of Comparison	two directly comparable familiar objects	to decide how the objects are the same or different (e.g., identify which is longer, heavier, or holds more, has more/fewer sides)	use comparative language for each type of comparison
CCR.M.GEO.PK.F	[EXT] Measuring time/duration	an activity	to measure how much time has passed	select a unit for measuring time (e.g., counting, or using a sand timer)
CCR.M.GEO.PK.E	[EXT] Indirect comparisons	two familiar objects that cannot be obviously compared	to describe what is unique about the object	select a method for contrast, and explain reasoning in everyday language

Code	Name	Given	Asked	Can
CCR.M.GEO.PK.D	Direct comparisons	two directly comparable familiar objects	to decide which is longer, heavier, or can contain the other	follow a systematic process to compare them and use comparative language to explain their findings
CCR.M.ARI.K.A	Compare collections to 100 with partitioning	collections of objects between 20 and 100 objects	to compare the number of objects in each collection	recognize that there are strategies more efficient than counting 1 by 1 to compare groups, and use them, e.g., count each collection by putting them in same size groups, counting the groups, and adding the remainder among others or stack elements of the collection and measure the resulting heights against each other, compare weights, etc.
CCR.M.ARI.K.J	Concrete representation of sharing	a practical equal sharing (division) situation represented verbally and a set of (subdivided, if applicable) manipulatives	to create a physical representation of this sharing situation	represent the allocation with concrete materials, and describe their solution in informal language using dealing (for collections) and partitioning (for wholes)
CCR.M.ARI.K.G	Adding and subtracting numbers by counting on	an addition or subtraction number sentence or authentic real-world problem	to solve the problem by counting	add or subtract the numbers by comparing the two groups, choosing the larger one to start with for addition or the smaller for subtraction, and counting up

Code	Name	Given	Asked	Can
CCR.M.ARI.K.H	Addition is Commutative	an addition number sentence or authentic real-world problem	to solve the problem by counting	add the numbers by comparing the two groups, choosing the larger one to start with, and counting up
CCR.M.GEO.K.K	Measure duration	starting and ending time(s)	to measure duration (e.g., using a stopwatch or counting on a calendar)	express duration in context using months, weeks, days, hours, minutes, and seconds
CCR.M.GEO.K.F	Uniform informal unit	a large object	to measure it with informal units	select an informal unit for measuring and express the answer in terms of those units (e.g., find its length using paperclips, volume using smaller containers, or weight using balance scale and other items)
CCR.ALG.K.B	Skip counting patterns	a beginning number and a difference to skip count by	to describe number patterns formed by skip-counting	generate the set of numbers by skip counting and note the repeating pattern in the ones digit
CCR.ARI.K.N	Patterns in counting by 10s	different starting numbers	to describe the number pattern formed by adding ten at a time from any starting point	describe the incrementing pattern in the tens digit (0 to 9)
CCR.ARI.K.M	Count by small groups	a large number of objects that would be tedious to count one by one	to find how many	count a large number of objects by quickly counting small groups e.g., 2s, 3s, 4s, or 5s.

Code	Name	Given	Asked	Can
CCR.M.SP.K.C	Represent data 1:1 with objects and drawings	a data set	to express the data mathematically	represent data with objects and drawings where one object or drawing represents one data value and describe the displays
CCR.M.ARI.1.N	Expressing 8ths and 4ths through repeated halving	a representation of one whole or a collection divisible by 8	to represent quarters and/or eighths through repeated halving	separate the collection or partition the whole into quarters and then eighths
CCR.M.ARI.1.L	Division by dealing	a mathematical or authentic real-world problem requiring evenly allotting a total number of objects equally between a given number of sets (with no remainder)	to find the number of items in each of the sets	recognize and represent the method of grouping by allocation and solve simple problems using these representations
CCR.M.ARI.1.M	Division using arrays (partitioning)	a scenario involving evenly dividing a total number of objects into sets of a given size	to represent dividing the total by the size of the set with an array	solve simple problems by creating an array and describe the division relationships between the subgroups and the whole collection represented by the array
CCR.M.GEO.1.C	Choosing and using informal units	one or more measurable things	to measure and compare measurements	Identify an appropriate informal unit and use it to measure and compare a given attribute (e.g., length, coverage, capacity, time)
CCR.M.ALG.1.A	Number patterns / sequences on a	a starting point and an incremental value	to represent this pattern on a number line	represent patterns on a number line

Code	Name	Given	Asked	Can
	number line			
CCR.M.ARI.2.D	Fractions as Equal subdivisions of a whole	a scenario involving equal divisions of a whole presented as a variety of shapes	to express symbolically and visually	express symbolically and visually unit fractions including 1/2, 1/3, 1/4, 1/5 and explain how many equal pieces make one whole for each fraction
CCR.M.ARI.2.B	Choosing strategies for Multiplication and Division with no remainder	an authentic real-world problem involving multiplication and/or division with no remainder	to solve and explain your answer	represent visually, then choose and apply efficient mental and written strategies and use appropriate digital technologies to solve
CCR.M.ARI.2.E	Fractions as equal subdivisions of a collection	a scenario involving equal divisions of a collection	to express symbolically and visually	express symbolically and visually unit fractions including 1/2, 1/3, 1/4, 1/5 and explain how many equal portions make one collection
CCR.M.GEO.2.F	Standard units	one (or more) objects	to measure (or measure and compare) length, mass and/or capacity in standard units	measure, order and compare objects using standard units of length, mass and capacity
CCR.M.GEO.2.E	Time differences to the minute	two times, rounded to the nearest minute	to find how much time passed	calculate the difference and present the time in minutes and hours

Code	Name	Given	Asked	Can
CCR.M.GEO.2.G	Total measurements	measurements in a common unit (e.g. segments of a path, weights of components) in the context of an authentic scenario	to find the total value of the set (or subset) of measurements	identify the relevant measurements and add up the measurements to calculate the total and apply appropriate units
CCR.M.ARI.3.K	Multiplication facts up to 10x10	mathematical problems presented verbally or symbolically involving multiplication or division of whole numbers to 10	to solve and explain your answer	recognize mathematical notation or language for multiplication or division, recall multiplication facts up to 10 × 10 and use them to solve the multiplication or division problem
CCR.M.ARI.3.D	[LO] Equivalent Fractions in context	an authentic real-world problem involving proportional reasoning where the simplified fraction is a unit fraction	to identify the fraction and express it as in equivalent ways	express equivalent fractions and explain them in context
CCR.M.ARI.3.J	Adding and Subtracting Equal Unit Fractions	an authentic real-world problem involving adding and/or subtracting two or more unit fractions of the same size	to express the problem verbally, symbolically, or visually and to find the total	add or subtract unit fractions and express the solution as a fraction or mixed number
CCR.M.ARI.3.G	Fractions on a Number Line	a unit fraction that is either a half, a third, or a quarter	to represent the fraction	locate quarters, halves, and thirds on a number line or represent on a unit bar and explain how many equal pieces make one unit for each fraction

Code	Name	Given	Asked	Can
CCR.M.ARI.3.N	Estimating multiplication and division of large numbers	a mathematical or an authentic real-world problem involving multiplication or division of large numbers	to estimate the answer quickly using mental arithmetic	use mental strategies to estimate; for example, rounding and using a multiplication fact to arrive at a reasonable estimate (e.g., 493/5 → 500/5 = 100)
CCR.M.GEO.3.E	Area formulas rectangle, parallelogram, triangle	a square, rectangle, parallelogram or triangle with related base and height measurements (in any orientation)	to find the area of the given figure	use the Area = Base * Height formula to determine the area of the square, rectangle, or parallelogram; recognize that two copies of a triangle can be shaped into a parallelogram with the same base and height to develop the formula Area = (1/2)*Base*Height
CCR.M.GEO.3.B	Compare areas	two 2D (regular or irregular) geometric figures and sufficient measurements or a scale	to compare the areas	compare the area of regular and irregular shapes using informal or formal units of area, inclusion, and/or covering
CCR.M.DC.4.E	Creating an algorithm for multiplying by a fraction	a scenario with a problem involving multiplying by a fraction	to explain how to solve the problem and why	articulate an algorithm to solve the problem
CCR.M.DC.4.D	[LO] Multiplying fractions	a problem involving multiplying two fractions with or without context	to find the product	multiply fractions

Code	Name	Given	Asked	Can
CCR.M.ARI.5.H	Add/Subtract general fractions	an authentic real-world problem involving addition and/or subtraction of fractions (incl. with unlike denominators)	to solve, interpret the results, and justify the reasoning	to write or verbalize an equation that represents the problem, solve, and interpret solutions in context
CCR.M.ARI.5.D	Rational numbers	a real number	to identify whether the given number is rational	identify rational numbers as any number that can be expressed as a ratio of integers, p/q, q ≠ 0
CCR.M.GEO.5.C	Area of quadrilaterals	quadrilaterals (e.g., kites, rectangles, squares, parallelograms, trapezoids, rhombuses) with a sufficient number of measurements	to find the area	decompose quadrilaterals into triangles and rectangles, as necessary, and compute the area
CCR.M.GEO.5.F	Volume of prisms	a right prism with indicated measurements for area of the base and the height of the prism	to find the volume	apply Volume = (Area of Base) * Height for right prisms
CCR.M.GEO.5.G	[EXT] Volume of non-prisms	a 3D solid with identical parallel cross sections	to find the volume	apply Volume = (Area of Base) * Height
CCR.M.DC.6.B	Identifying patterns for compression	data with some repeating sequences embedded in it	to compress the data using pattern-replacement	recognize the repeating pattern and use pattern-replacement to compress the data and explain their algorithm

Code	Name	Given	Asked	Can
CCR.M.GEO.6.G	Capacity vs Volume	a familiar, hollow 3D container	to find a measure of capacity using liquid or objects and compare it to the calculated volume of the container	recognize that the smaller the objects (liquid being the smallest), and thinner the sides of the container, the closer the capacity is to the volume
CCR.M.GEO.6.I	Estimating the volume of 3D solids	a 3D geometric solid representing a real-world object	to estimate the volume	estimate the volume of the solid by embedding in a standard prism
CCR.M.GEO.6.H	Areas of circles	A circle and measuring tools	to decompose the circle into approximate known shapes to find the area	apply a consistent smaller shape (e.g., using graph paper) to determine the approximate area and verify that it closely matches the area as calculated by pi*r^2. Recognize that the smaller the shape, the closer the estimate is to the true area.
CCR.M.GEO.7.A	Surface areas and volume of standard shapes	a diagram, graphical representation, or model of 3-dimensional solid (e.g., right pyramid, cone, cylinder, prism, sphere) or a composite solid in a mathematical or authentic problem and a formula reference sheet or access to research tools	to find the volume and/or surface area	find the volume and/or surface area by decomposing the solid and applying known formulas, applying proportional reasoning as needed

Code	Name	Given	Asked	Can
CCR.M.ARI.8.A	Significant Figures informally	two or more measurements in an authentic real-world context	to arithmetically combine the measurements and interpret in context	recognize the importance of error and reduce the precision in their final result, as applicable within the context so as not to imply a greater level of precision than is warranted