Implementing the Proposed Mathematics Framework: Recommendations for PISA 2021

Peggy G. Carr, Ph.D., Vice Chair, PISA Governing Board
Associate Commissioner, National Center for Education Statistics (NCES)
May 25, 2018
The PISA 2021 Mathematics Framework

Based on

- 2012 Mathematics Framework (minor modifications in 2015 and 2018)
Everyday Experience with Mathematics is Changing
Comparing Skills of Computers and Adults

Percentage of Adults Ages 16-65 by PIAAC Numeracy Proficiency Levels

- Level 1 and Below: 27%
- Level 2: 34%
- Level 3: 29%
- Level 4-5: 10%

SOURCE: Organization for Economic Cooperation and Development (OECD), Program for the International Assessment of Adult Competencies (PIAAC), 2012
2012 PISA Mathematics Framework

Figure 1.1
A model of mathematical literacy in practice

Challenge in real world context
Mathematical content categories: Quantity; Uncertainty and data; Change and relationships; Space and shape
Real world context categories: Personal; Societal; Occupational; Scientific

Mathematical thought and action
Mathematical concepts, knowledge and skills
Fundamental mathematical capabilities: Communication; Representation; Devising strategies; Mathematisation; Reasoning and argument; Using symbolic, formal and technical language and operations; Using mathematical tools
Processes: Formulate; Employ; Interpret/Evaluate
2022 PISA Mathematics Framework

Figure 1.1
A model of mathematical literacy in practice

Challenge in real world context
Mathematical content categories: Quantity; Uncertainty and data; Change and relationships; Space and shape
Real world context categories: Personal; Societal; Occupational; Scientific

Mathematical thought and action
Mathematical concepts, knowledge and skills
Fundamental mathematical capabilities: Communication; Representation; Devising strategies; Mathematization; Reasoning and argument; Using symbolic, formal and technical language and operations; Using mathematical tools
Processes: Formulate; Employ; Interpret/Evaluate

Problem in context
Formulate
Mathematical problem
Employ
Mathematical results
Evaluate
Interpret

Formulating
Reasoning
Employing
Interpret & evaluate
Defining Mathematical Literacy in 2021

Mathematical literacy is an individual’s capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts.

It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens.
Preserving the underlying problem solving processes ensures that trend is maintained while the framework is expanded to include reasoning.
Current PISA mathematics subscales

Additions to PISA math subscales for 2021
- Geometric approximation
- Growth phenomena
- Conditional decision making
- Computer simulations

Current PISA mathematics subscales
- Challenge in a real world context
- Reasoning
- Space and shape
- Change and relationships
- Uncertainty and data
- Quantity
- Formulating
- Employing
- Interpret & evaluate

Contexts
- Personal
- Occupational
- Societal
- Scientific
Number systems and algebraic properties

Mathematics as a system based on abstraction and symbolic representation

Structure of mathematics and its regularities

Functional relationships between quantities

Mathematical modeling as a lens to the real world

Variance at the heart of statistics

Addition of PISA Reasoning
21st-century skills relevant to math

- Critical thinking
- Research and inquiry
- Creativity
- Reflection
- Initiative
- Information use
- Systems thinking
- PERSISTENCE
- Communication
- Self-direction
Broadened Model for Mathematics

**Challenge in Real-world Context**
- Mathematical content categories:
  1. Quantity (computer simulations)
  2. Uncertainty and data (conditional decision making)
  3. Change and relationships (exponential growth)
  4. Space and shape (geometric approximation)
- Real world context categories: Personal, Societal, Occupational, Scientific

**Mathematical Reasoning and Problem Solving**
- Mathematical concepts, knowledge and skills
- Fundamental concepts supporting mathematical reasoning
  1. Number systems and their algebraic properties
  2. Mathematics as a system based on abstraction and symbolic representation
  3. The structure of mathematics and its regularities
  4. Functional relationships between quantities
  5. Mathematical modeling as a lens to the real world (e.g. those arising in the physical, biological, social, economic, and behavioral sciences)
  6. Variance as the heart of statistics
- Processes: Formulate, Employ, Interpret/Evaluate
- 21st century skills specifically relevant to mathematics:
  1. Critical thinking
  2. Creativity
  3. Research and inquiry
  4. Self-direction, initiative, and persistence
  5. Information use
  6. Systems thinking
  7. Communication
  8. Reflection

*Note. The mathematical content category topics listed in parentheses are subtopics from each of the content categories that should receive greater emphasis given their relevance to important societal issues and the nature of the new economy.*
Translating the 2021 Framework into An Assessment
Take Advantage of the Digital Delivery System

Digital-based assessment (DBA) allows for measurement of skills and processes that cannot be accomplished through traditional paper-and-pencil assessments.

Examples

• Allows for simulations, which with captured log data, may be used to measure processes of mathematical reasoning.
Sample Reasoning Item from 2012

CD PRODUCTION

Zodiac provides a CD copying service. There are two methods for making copies of CDs - duplication and replication.

The graph and the price calculator show the prices for copying different numbers of CDs using the two methods. You can enter different values in the ‘Number of copies’ cell to find the exact cost of duplication and replication.

Question 3: CD PRODUCTION CM01-SQ02

Zodiac makes the following statement in its advertising: Duplication is cheaper for short-run copying (up to 500 CDs).

Explain why the number, 500 CDs, in the statement is incorrect.

What is the maximum number of copies which would make the statement correct?

Number of copies =
Sample Reasoning Item from 2012
Take Advantage of the Digital Delivery System

Digital-based assessment (DBA) allows for measurement of skills and processes that cannot be accomplished through traditional paper-and-pencil assessments.

**Examples**

- Allows for simulations, which with captured log data, may be used to measure processes of mathematical reasoning.
- Allows for *adaptive testing*, which tailors the selection of items for students.
Adaptive Testing: What is it?

• In traditional large-scale assessments, items are administered to each student in an intentionally randomized order.

• In adaptive testing, items are selected based on each student’s performance so that he or she gets items within his or her range of ability.

• Relies on a significantly larger item pool.
Elements of the PISA adaptive design

- **Items** – various item types including machine-scored items, human-coded items
- **Testlet** – a set of several units, smaller than the cluster; the same unit can appear in multiple testlets.

**Core**
(8 testlets of medium difficulty)

**Stage1**
(16 testlets)
8 High / 8 Low

**Stage2**
(16 testlets)
8 High / 8 Low
Illustration of 4 possible paths starting with the “Core1” testlet.
Example of how items match distribution of U.S. students

SOURCE: Rutkowski, L. (April 2018). Presentation to the National Academy of Education
Adaptive Testing: What does PISA gain?

• Greater fairness
  – Fewer students are unnecessarily exposed to a test that is too difficult for them to complete

• Greater consistency
  – The same test can be administered in all countries, overcoming the need for country-level adaptations

• Better measurement
  – Reduced measurement error for high- and low- performing students
Potential Threats to the Integrity of the Assessment

1. Vulnerable to drifting into a different construct if lower-level difficulty items are *too* low level
   - Need empirical research to know how much PISA can lower item difficulty without compromising the construct stability
Example of how items match distribution of high-level abilities of Finnish students

SOURCE: Rutkowski, L. (April 2018). Presentation to the National Academy of Education
Example of how items match low-level abilities of students in Dominican Republic

SOURCE: Rutkowski, L. (April 2018). Presentation to the National Academy of Education
Potential Threats to the Integrity of the Assessment

1. Vulnerable to drifting into a different construct if lower-level difficulty items are *too* low level
   - Need empirical research to know how much PISA can lower item difficulty without compromising the construct stability

2. Construct is at risk of being altered if new and enhanced DBA items greatly outnumber trend items in DBA-format
   - Empirical research has shown that too many new DBA-enhanced items added to the item pool can put trend lines in jeopardy
Potential Threats to the Integrity of the Assessment

1. Vulnerable to drifting into a different construct if lower-level difficulty items are too low level
   • Need empirical research to know how much PISA can lower item difficulty without compromising the construct stability

2. Construct is at risk of being altered if new and enhanced DBA items greatly outnumber trend items in DBA-format
   • Empirical research has shown that too many new DBA-enhanced items added to the item pool can put trend lines in jeopardy

3. Vulnerable to compromising construct validity if DBA items are scaled with paper and pencil items without equating
   • Need empirical research on mode effects and how to scale data together
Thoughts on the Possibility of Achieving these Goals

• In technologically evolving societies, changes in assessments are inevitable and necessary to stay relevant.
• Recognizing the constant need for change is an integral part of good assessment development.
• Building into the system R & D for transitioning to new approaches (whether methodologies, frameworks, or modalities) can preserve trend while staying relevant.
Questions?
Implementing the Proposed Mathematics Framework: Recommendations for PISA 2021

Peggy G. Carr, Ph.D., Vice Chair, PISA Governing Board
Associate Commissioner, National Center for Education Statistics (NCES)
May 25, 2018