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Opinion Short Paper

What "Mathematics of AI" should be taught in schools?

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As Artificial Intelligence (AI) gains attention on a global scale, apprehension grows, fueled by misconceptions and fears based on a lack of understanding. It is possible to educate our youth to better understand a world with AI by equipping them with and demystifying the mathematics needed to understand its conceptual processes.

In his book *The Master Algorithm,*[1] Pedro Domingos outlines five "tribes" that use different algorithmic processes in their various approaches to AI. The "tribes" and the mathematics needed to understand them are outlined in the table below:

"Tribe"	General Approach	Mathematics Required
Symbolists Decision Trees	Understand deductive logic and work backwards to determine a premise from the facts	Logical Deduction Induction Best Fit/Regression Statistical Significance
Connectionists Neural Networks	View knowledge from a lens of neural connections, and attempt to simulate the workings of the brain	Classification and pattern recognition Slopes and tangent lines Optimization Matrix multiplication and transformation Basic and conditional probability Regression
Evolutionaries Genetic Algorithms	Mimic evolution and survival of the fittest	Basic probability and combinatorics Function fitting Optimization
Bayesians Naive Bayes Classifier	Create assumptions and update prior beliefs with evidence of new information based on data	Basic and conditional probability Bayes Theorem Markov chains and Bayesian networks Graph Theory

[1] https://en.wikipedia.org/wiki/The_Master_Algorithm "The Master Algorithm". Basic Books 2015

Analogizers Nearest Neighbor	Identify how similar one idea is to another and infers learning	Represent and interpret data Coordinate plane Association in bivariate data Euclidean Distance formula Error and cost function types Differentiability

The discussion below examines where this content can currently be found, and where it might be found in the future.

By examining the US Common Core State Standards for Mathematics (CCSS-M) and the UK General Certificate of Secondary Education (GCSE), the following topics were noted as already taught by both sets:

- Starting at a young age, students are asked to classify objects and recognize patterns, which is at the core of what many algorithms do.
- Plotting points, representing data in different ways, and interpreting both one and two variable data are common in the middle grades, along with the use of the euclidean distance formula to find the distance between two points.
- Logical deduction is taught in the context of algebraic and geometric proof with concrete objects.
- Regression is covered deeply for linear fit, and minimally with non-linear models.
- Basic probability is included in most standards, but (in the US) is often marginalized and left out as time is tight towards the end of the school year and the content isn't viewed as foundational for the skills of the next year.

Some topics are included in standard sets, but listed as optional or only for advanced students:

- Matrices, which are key to representing data and operations in machine learning algorithms, are included for advanced students in the CCSS-M.
- Additionally, using probability to make decisions, another key feature of machine learning, is an additional topic in the CCSS-M, along with Bayes theorem.
- GCSE mathematics requires knowledge of vectors, which could be easily extended to matrices.
- GCSE mathematics include the concept of instantaneous rate of change and conditional probability for advanced students only.

The Center for Curriculum Redesign (CCR) has, jointly with the Australian Curriculum and Assessments Authority (ACARA), designed standards[2] (aka Curriculum) for grades K-9 which address some of the content gaps:

- Statistical significance is addressed conceptually in grade 9.
- Logical statements with AND, OR and XOR operators are addressed in grade 7. Research[3] psychologist Jean Piaget indicates that students are cognitively ready for logic with concrete objects by ages 7-10, and more formal logic by ages 11-15.
- Conditional and Bayesian reasoning is developed in grades 5-7, and requires students to use probability to inform decision making.
- Graph theory is included in grade 9, which lays the foundation for Bayesian networks.
- Probabilities[4] also cover Logic, Causality (and Psychology/Decision-making), not just simple, compound, and combinatorial situations.

Absent from all of these frameworks were certain skills that belong in high level, specialized high school courses, or at the college level. These include:

- Logic gates are more often included in computer science classes, and this concept could be understood by all students.
- Calculus provides the foundation for optimization used in machine learning, and the skills of differentiation and integration should remain in this specialized class; however, the concepts of optimization and slope can be taught in the standard K-12 program.
- Statistics classes teach students to apply different tests of statistical significance, and remains a specialized skill, but the concept of statistical significance can be incorporated earlier.
- Markov Chains should be incorporated into a class on linear algebra, rather than taught with matrices in an Algebra II course due to the number of dimensions that would arise within a matrix to represent a Markov Chain of sufficient complexity. Computational models would be best suited to analyze and compute with these matrices.
- Bayesian Networks combine the concepts of graph theory and probability, and would be addressed in a college level programming course, although foundational concepts of both graph theory and probability can be taught in the standard K-12 program.

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[2] https://curriculumredesign.org/modern-mathematics/

[3] Piaget, J. (1972). Intellectual Evolution from Adolescence to Adulthood. *Human Development, 51*(1), 40-47. DOI: 10.1159/000112531

[4] https://curriculumredesign.org/wp-content/uploads/Why-are-even-basic-Probabilities-so-difficult-to-learn-Charles-Fadel-CCR.pdf

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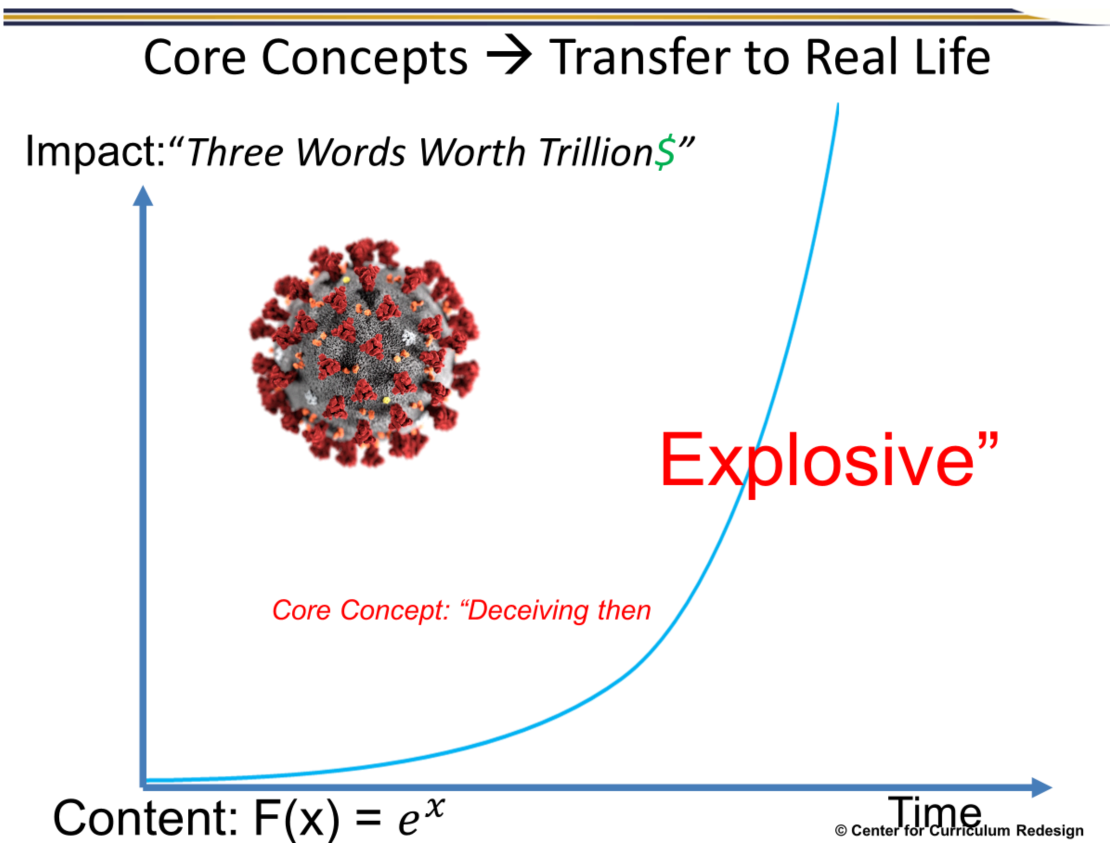
Content vs Concept: a lingering confusion in Mathematics

In its work with ACARA, CCR has spent three years making explicit the difference between the two, which, from experience, are almost always conflated in Mathematics (unlike Humanities disciplines like History). This has led to the CCR publication of "Core Concepts in Mathematics"[5] at the Discipline and Branch level. A telling example is exponentials, where the formulation $f(x) = e^x$ or equivalent is never understood by students (and therefore eventually adults in positions of power) as "deceiving then explosive."



*It is critical to point out that learning concepts is **NOT** in replacement of content.* The two are complementary and must be fused together whenever possible. However, depending on the grade level and the student's mathematical abilities, the concepts may be all that they are able to understand and retain (which is valid for all students, STEM-bound or not). Only the STEM-bound students may be exposed to the mathematical formalism of some of the more complex topics such as matrices.

[5] https://curriculumredesign.org/wp-content/uploads/Core-Concepts-in-Mathematics-DCC-BCC-CCR.pdf

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RECOMMENDATIONS:

As shown above, a reasonable amount of "Math for AI" is already covered, albeit improperly (optional, dropped out, only for STEM-bound, etc.). This can be remedied by:

1. **Embracing Statistics/Probabilities, and Discrete/Computational** mathematics with the same vigor as the traditional branches of Arithmetic, Geometry and Algebra, in K-9. This will require the courage to:
    - Redesign the standards/curricula (already done by CCR)[6]
    - Devote significant professional development for teachers (CCR offers courses[7] on 6 critical topics: exponentials, basic/combinatorial probabilities, Bayesian probabilities, algorithms and graph theory, game theory, and complex systems).
    - ***Most critically, modify the university entrance requirements to allow[8] such adaptation!***
2. Differentiating between Concept-level vs content-level of understanding, depending on ALL vs STEM-bound students. (Concepts for all, Content as much as the student can handle). This may be considered anathema by some mathematicians, who would seemingly rather sacrifice understanding of implications for real-world contexts to algebraic formalism.
3. Addressing **conceptually** the following topics for **ALL** students:
    - Optimization: can be included as early as elementary school by thinking about where a ball will land in a 2D or 3D model.
    - Instantaneous slope of the tangent line: can be included with the introduction to different types of functions by asking students to consider where a graph is increasing or decreasing the fastest or slowest; Where is it steepest? Where is it flattest?
    - Statistical significance: can be included conceptually along with statistics and probability in high school by asking students to consider, "if we step back and think about what we observed, is it realistic to think this was more than just random chance?"
    - Matrix multiplication and transformation: can be included in a discussion after geometric transformations of points, with matrices viewed as an extension of the same ideas.

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[6] https://curriculumredesign.org/modern-mathematics/
[7] https://4dedu.org/mathematics/
[8] https://curriculumredesign.org/wp-content/uploads/Inertia-in-Education-CCR-Final.pdf