CENTER FOR CURRICULUM REDESIGN

Mathematics for the 21st Century

Paper #1

WHAT should students learn?

Branches, Subjects and Topics

Prepared by Charles Fadel, Center for Curriculum Redesign

July 23, 2014

With sincere thanks, and appreciation for its generous support, to the:

Fondation
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Introduction

In the 21st century, humanity is facing severe difficulties at the societal economic and personal levels:

- Societal:
    - Greed manifested via financial meltdown, global warming, and personal privacy invasions.
    - Intolerance manifested as religious fundamentalism and political absolutism.
- Economic: Challenges such as globalization, and the pursuit of innovation.
- Personal: Goals of employability, and happiness.

Technology's exponential growth is rapidly compounding the problems via automation and offshoring, which are producing social disruptions. Education is falling behind the curve[1], as it did during the Industrial Revolution.

Profound changes to curriculum[2] were effected in the late 1800's as a response to the sudden growth in societal and human capital needs. As the world of the 21st century bears little resemblance to that of the 19th century, education curricula are overdue for a major redesign.

The Center for Curriculum Redesign's mission is to answer the seminal question: "*What* should students learn for the 21st Century?™ " and the question is quite acute in Mathematics. In 2013, the Center conducted a conference in Stockholm, Sweden and a colloquium in East Hampton, NY, USA focused on Mathematics. OECD countries spend $236B ***per year*** on Mathematics education[3] yet most countries report shortages in Science, Technology, Engineering and Maths (STEM) talent.

Why Teach Mathematics?

From Aristotle, Plato, Al-Khawarizmi, and Al-Kindi, to John Allen Paulos (Temple U.), Paul Ernest, (U. of Exeter), and Eleanor Robson (U. of Oxford), Maths thinkers have stated three types of reasons:

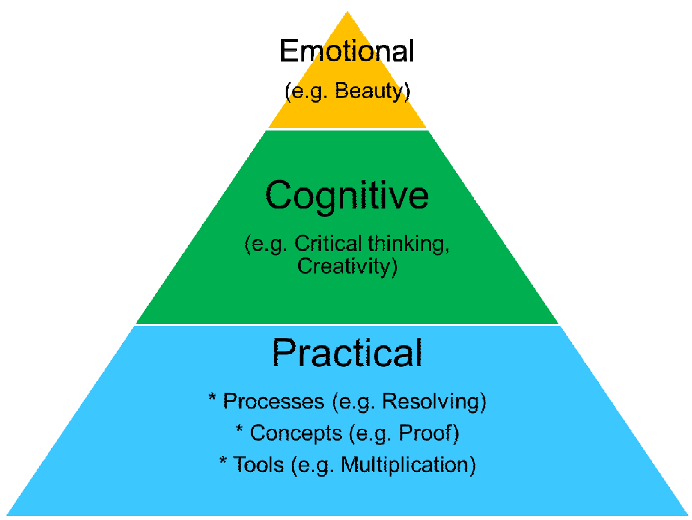
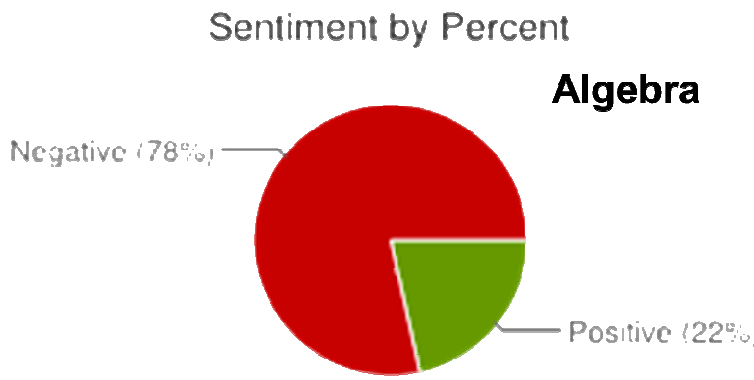


Figure 1: The three types of benefit used as justification of the mathematics curriculum. Source: Charles Fadel, CCR

[1] Goldin/Katz "The race between education and technology"
[2] Aka "standards", "programmes" etc depending on the country
[3] Private communication with Mr Andreas Schleicher, OECD Education Directorate

1. Emotional reasons: Like all disciplines, Mathematics has an intrinsic beauty that Mathematicians attempt to convey. Yet liking of Mathematics is anecdotally low among the general population, as exemplified by sentiment analysis of Twitter feeds, and cartoons



"In 1953 you were my math teacher. You promised that algebra would come in handy someday. How much longer do I have to wait?"

Insufficient efforts are made to insure a positive emotional response – the affective side of Mathematics that so often discourages many from pursuing the discipline with enthusiasm. This rejection starts very young, with parents passing down their attitudes both implicitly and explicitly to their children.[4] Much mathematics education fails to highlight the beauty of the subject to its students. Take for instance "recreational" mathematics: while geometric transformations are taught everywhere, tessellations (or origami) are usually segregated to "enrichment" activities. The construction of complex tessellations is a task that not only gives a student a sense of satisfaction due to the beauty of the finished design, but also requires the student to engage in perseverance and attention to detail to accurately reproduce the figures so the tessellation is correct.

[4] Poffenberger, Thomas, and Donald A. Norton. "Factors determining attitudes toward arithmetic and mathematics." The Arithmetic Teacher 3.3 (1956): 113-116.

2. Cognitive reasons: Mathematics is most often associated with the development of Critical Thinking and broad reasoning abilities. Yet research[5] by the CCR has shown that this connection is not proven. It states: "…from the current findings it is most likely that higher order functions support the building of mathematics skills, not the reverse… We have shown here that there is not sufficient evidence to conclude that mathematics as they are taught now enhance higher order cognitive functions."

3. Practical reasons: Mathematical understanding is crucial for high performance in our personal, public, and work lives. At home, we may want to understand the results of a medical test, or rekindle our child's love of math. As citizens, we may want to judge the rise in carbon-dioxide levels in the air, or the proportion of tax dollars that should go to health, education, or war. At work, we may need to estimate the money, time, and employees for a large project. Finally, mathematics underlies our science, technology, and engineering.

Maths for all current and new Occupations:

The needs for different types of Mathematics have been documented by several reports:

1) The OECD's Global Science Forum Report on Mathematics in Industry (July 2008) described the following needs:

Themes	Responses
	Knowledge
Complexity	Complex systems
Uncertainty	Statistics & probabilities
Multiple scales	Complex systems
Simulations & Modeling	Computational maths (algorithms)
Data & Information	Statistics & probabilities
	Skills
Multidisciplinarity	Collaboration
Transfer of knowledge	Communication

2) The Royal Society's ACME 2011 "Mathematics in the workplace and higher education" describes the following requirements:

- Mathematical modeling (e.g. energy requirement of a water company; cost of sandwich; …)
- Use of Software, and coping with problems (e.g. oil extraction; dispersion of sewage; …)

[5] See "Does Maths education enhance Higher-Order Thinking Skills?" by CCR

- Costing (allocation, dispute management) (e.g. Contract cleaning of hospital; management of railway; …)
- Performance and ratios (e.g. Insurance ratios; glycemic index; …)
- Risk (e.g. clinical governance; insurance; …)
- Quality/SPC control (e.g. Furniture; machine downtime; deviation of rails; …)

3) The Freudenthal's Institute Arthur Bakker presented at the CCR's Stockholm conference:

- "Most important in many occupations are
  - Number, quantity, measure
  - Data handling and uncertainty
- Followed by
  - Space and shape
  - Relations, change, formulas"

4) Pr. Michael Handel's STAMP[6] survey showing the relative use of various branches of Maths by class of occupation: (WC = White Collar; BC = Blue Collar)

	All	Upper WC	Low WC	Upper BC	Low BC	Service
Percentage (weighted)	100	36.1	25.4	10.3	13.0	15.1
N (unweighted)	2,304	1,010	569	161	271	291
Percentage using:						
1. Any math	94	95	97	94	91	88
2. Add/subtract	86	93	90	87	78	73
3. Multiply/divide	78	89	82	81	65	57
4. Fractions	68	82	68	70	58	40
Any more advanced	22	35	9	41	19	4
5. Algebra (basic)	19	30	8	36	16	4
6. Geometry or trig	14	20	5	29	15	2
7. Statistics	11	22	5	10	6	2
8. Algebra (complex)	9	14	3	16	8	2
9. Calculus, similar	5	8	1	8	5	1

[6] "What do people do at work? A Profile of U.S. Jobs from the Survey of Workplace Skills, Technology, and Management Practices (STAMP)"; Michael J. Handel OECD and Department of Sociology, Northeastern University; 20 June 2010

5) The CCR's analysis of Maths usage by occupations shows:

Occupation (below)	Algebra	Applied Maths	Calculus	Discrete Mathematics	Foundations	Geometry	Numbers & Operations	Statistics & Probability	Topology & Recreational
Taxonomy & Ontology: Wolfram Research →	Matrices, Operations, Vectors etc	Complex systems, Control, Game theory, etc	Analysis, Transforms, Polynomials, etc	Automata, Graphs, Computational maths etc	Sets, Logic etc	Curves, Dimensions, Transformations, Trigonometry, etc	Arithmetic operations, Fractions, Sequences, etc	Distributions, Analysis, Estimation, etc	Knots, Figures, Folding, Spaces, etc
Agriculture						X	X	X	
Architecture		X				X	X	X	X
Astronomy/Cosmology	X	X	X	X		X	X	X	X
Biology, Botany, Zoology		X		X			X	X	
Biotechnology, Genetics	X	X	X	X		X	X	X	X
Business		X					X	X	
Cinematography/Photography						X	X		X
Civil engineering	X	X	X	X		X	X	X	X
Communication		X					X	X	
Computer science	X	X	X	X	X	X	X	X	X
Craftsmanship						X	X		X
Dance						X	X		X
Design						X	X		X
Drawing						X	X		X
Economics & Finance	X	X	X	X		X	X	X	
Education	X	X	X			X	X	X	
Electrical engineering	X	X	X	X		X	X	X	
Environmental science	X	X	X	X		X	X	X	
Ethics							X		
Geography/Geology	X	X	X	X		X	X	X	X
Health							X	X	
History/Archeology	X	X		X			X	X	
Journalism	X	X					X	X	
Languages/Linguistics	X	X		X			X	X	
Law		X					X	X	
Materials Science/Nanotechnology	X	X	X	X		X	X	X	X
Mechanical engineering, Robotics	X	X	X	X		X	X	X	X
Medicine/Pharmacy/Veterinary		X					X	X	
Music	X						X	X	
Painting						X	X		
Philosophy		X			X		X	X	
Physics	X	X	X	X	X	X	X	X	X
Poetry/Prose							X		
Psychology/Sociology/Anthropology	X	X		X			X	X	
Sculpture						X	X		X
Sewing/Knitting/Tapestry						X	X		X
Spirituality/Religions							X		
Theater/Acting							X		X

Lastly, the CCR's East Hampton Colloquium's participants recommended the following additions/augmentations to Maths standards:

- Statistics & probabilities including visualization
- Discrete/algorithmic mathematics including modelling
- Applied Mathematics including complex dynamical systems, fractals etc.
- Estimation
- Logic and argumentation
- Programming
- Use of computer-based computation

And deletions:

- By-hand algebraic computation[7], including long division
- "Fake realism" of examples

The synthesis of all of the above highlights the need to rebalance traditional Mathematics (Geometry, Algebra, Calculus) with new branches[8] (Statistics & Probabilities; Applied Maths; Discrete Maths) which are relevant for a wide swath of occupations.

Shockingly perhaps, this is not particularly new! a ***1982*** US National Science Foundation report[9] stated that Mathematics education needed:

[7] Which echoes Conrad Wolfram's (Mathematica) TED talk "Stop Teaching Calculating, Start Teaching Math"
[8] Sometimes referred to as "Mathematical Sciences", which for the sake of this analysis is seen as an irrelevant academic distinction.

- “more emphasis on estimation, mental maths…
- “less emphasis on paper/pencil execution…”
- “content in… algebra, geometry, pre-calculus and trigonometry need to be… streamlined to make room for important new topics.”
- “discrete Mathematics, statistics/probabilities and computer science must be introduced”.

And to quote[10] Google’s Hal Varian, Chief Economist: “*“I keep saying the sexy job in the next ten years will be statisticians. People think I’m joking but who would’ve guessed that computer engineers would’ve been the sexy job of the 1990s.”*

Maths for Personal and Societal uses:

John Allen Paulos, Mathematician at Temple University, and Author of “*A Mathematician reads the newspaper*“ has stated: “*Gullible citizens are a demagogue’s dream… almost every political issue has a quantitative aspect*”.

Here again, PISA’s definitions of Personal and Societal uses encompass[11]:

- Personal: Mostly arithmetic and spatial:
    - Personal finance
    - Proportional reasoning
    - Understanding technical documents (plans, charts, etc)
    - Mental maths (percentages, four operations, estimating etc)
    - Estimation (as in measures/references/distances such as navigation, etc)
    - Basic geometry (billiards, parking, manual tasks, etc)
    - Spatial reasoning

- Societal: Related to data, logic, scale, chance, relationships:
    - Structured logical arguments
    - Understanding data: statistical
    - Chance/risk/uncertainty: probabilities
    - Visualization, presenting data
    - Magnitude of numbers (budgets, taxes, etc)
    - Rate of change (exp, log, S, etc)
    - Understanding Systems - scale (ecology, etc) incl. identifying relations between objects

Here again, Personal and Societal uses highlight the need to rebalance traditional Mathematics (Geometry, Algebra, Calculus) with new branches (Statistics & Probabilities; Applied Maths such as complex systems) and deepening the understanding of basic Arithmetic (number sense, proportionality).

[9] “What is fundamental and what is not” NSF 1982
[10] McKinsey Quarterly, Jan 2009
[11] http://www.oecd.org/pisa/pisaproducts/Draft%20PISA%202015%20Mathematics%20Framework%20.pdf

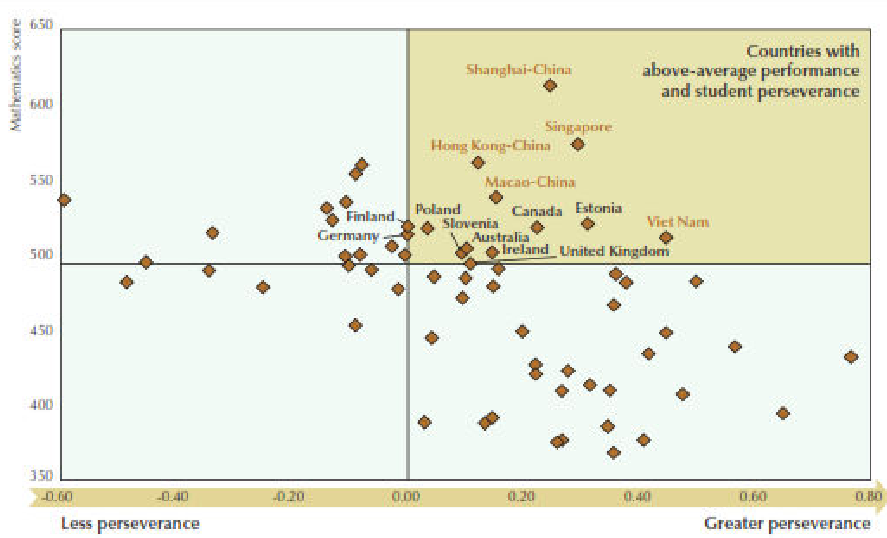
Skills, Character, Metacognition:

A- Skills: Of the "4 C's" of Skills[12], Critical Thinking and Creativity are most often mentioned in the context of Mathematics. Critical thinking is addressed in an accompanying paper[13]. To effectively exercise Creativity, CCR suggests a progression that builds up as:

1. Solve exercises (standard solutions)
2. Solve problems (standard solutions)
3. Solve problems using non-standard solutions (creative stretch)
4. Find in the real-world new problems, and solve using both standard and non-standard solutions
5. Create new problems, and solve using both standard and non-standard solutions
6. Create new *classes* of problems (metacognitive stretch) and explore solvability
7. Solve a *class* of problems

B- Character[14]: At the East Hampton colloquium, resilience/persistence was identified as potentially developed via Mathematics. Indeed, the OECD[15] has shown the relationship between student perseverance and Mathematics performance:

Performance in mathematics and student perseverance



Source: OECD, PISA 2012 Database, Tables I.2.3a and III.3.1d.
StatLink http://dx.doi.org/10.1787/888932935667
StatLink http://dx.doi.org/10.1787/888932963939

C- Metacognition: Reflection on the Processes used in inherent to Mathematics when taught appropriately, so please refer to CCR's companion paper "Maths Concepts and Processes".

Topics that should be covered elsewhere:

There is often a debate about whether uses of Mathematics such as personal finance and computer programming should be covered during Mathematics classes, to provide more context. Although

[12] www.p21.org
[13] "Does Maths education enhance Higher-Order Thinking Skills?" by CCR
[14] "Character" = "Agency, Attitudes, Behaviors, Dispositions, Mindsets, Personality, Temperament, Values" = "Social & Emotional Skills" (OECD)
[15] http://www.oecd.org/pisa/pisaproducts/pisainfocus/PISA-in-Focus-N37-(eng)-FINAL.pdf

CCR is completely in favor of contextual learning via real-world examples, it considers that both subjects are better taught as focus areas outside of Maths classes.

Personal finance indeed uses concepts such as compound interest, derivatives and the like, but the field of Finance is specialized enough to require a different teacher. Similarly, computer programming, although associated with structured logic, is also very specific and also requires a different teacher.

The importance of real-world contexts:

One of the key shortcomings of present-day Mathematics education has been the disconnection between applications and theory. CCR considers that Maths *must be connected with all its applied uses, not viewed separately.* Building a progression that goes from the concrete to the abstract is critical for motivation and eventually helps build student capacity to transfer the skills they learn to outside the classroom. This is true of learning in general; concepts are more likely to get internalized if they are built from the bottom up, eventually achieving the abstract rules, rather than imposed as abstract concepts from the top down[16].

The Role of Technology:

The role of Technology in Mathematics is multifold:

- As for other disciplines, it connects to a world of information and knowledge, which makes rote learning all the more unnecessary (minus some essential, carefully chosen exceptions such as multiplication tables for automaticity, etc.).
- It allows for deepening of the learning by freeing up time from routine hand tasks[17], and thus can help students focus on understanding the real-world problem at hand, reducing it to algebraic expression, and spending time analyzing the results.
- It enables, through the use of software tools[18], the exploration of new areas (Big Data; Fractals; Multivariable analysis; etc).

CCR views the use of computers, not just calculators, in Mathematics as essential as they are in the rest of modern life.

Conclusion: Our Aspiration – a more Numerate society

Are we teaching Maths for the 0.9% of students who end up as Maths majors, the 20-40% who end up studying STEM in college, or 100% of the population? If it is the latter, we need to rethink Maths standards with the following goals:

1. Processes AND Concepts AND Tools (balance)
    - Traditional AND modern Maths branches/subjects/topics (relevance)
    - Depth AND breadth (balance)
    - Application AND Formalism (balance and sequence)
    - Knowledge AND Skills AND Character AND Metacognition (balance)
    - Enabled by Technology (balance)

[16] See "Does Maths education enhance Higher-Order Thinking Skills?" by CCR
[17] See Conrad Wolfram's (Mathematica) excellent TED talk "Stop Teaching Calculating, Start Teaching Math"
[18] http://en.wikipedia.org/wiki/Mathematical_software

   - *Critically, it must be connected with all its applied uses, not viewed separately (context and motivation)*
2. Cognitive (transfer)
   - Skills: Creativity; critical thinking; communication; collaboration
   - Character: Resilience (persistence, confidence, resourcefulness)
   - Metacognition: learning how to learn; self-directed learning
3. Appreciation
   - Fun, beauty
   - If not love, at least curiosity not fear!

The CCR's Stockholm Declaration has stated:

"We call for a far deeper and reconceptualized understanding of Mathematics by the entire population as a critical right, requiring:

- a new vision of mathematics education that anticipates needs and reinforces the role of mathematics in society, economies, and individuals, and strengthens gender equity,

- changes to existing Mathematics standards as presently conceived, through a significant rethinking of what branches, topics, concepts and subjects should be taught in Mathematics for human, economic, social and career development,

- more inclusive assessments, including for example, cross-curricular competencies at the global (PISA, TIMMS), academic entrance (SAT, ACT, etc.), local (jurisdiction-specific) and classroom-level (formative and summative) levels, and providing data and information that can be used to help improve mathematics education at global and local levels.

- mobilization of public awareness through the media, and involvement of private and public sectors, governmental bodies, students, international organizations, and others in strengthening partnerships and networks for mathematics education, and in improving mathematics education globally."

Humanity has a very large stake in making these goals happen, and very soon.

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