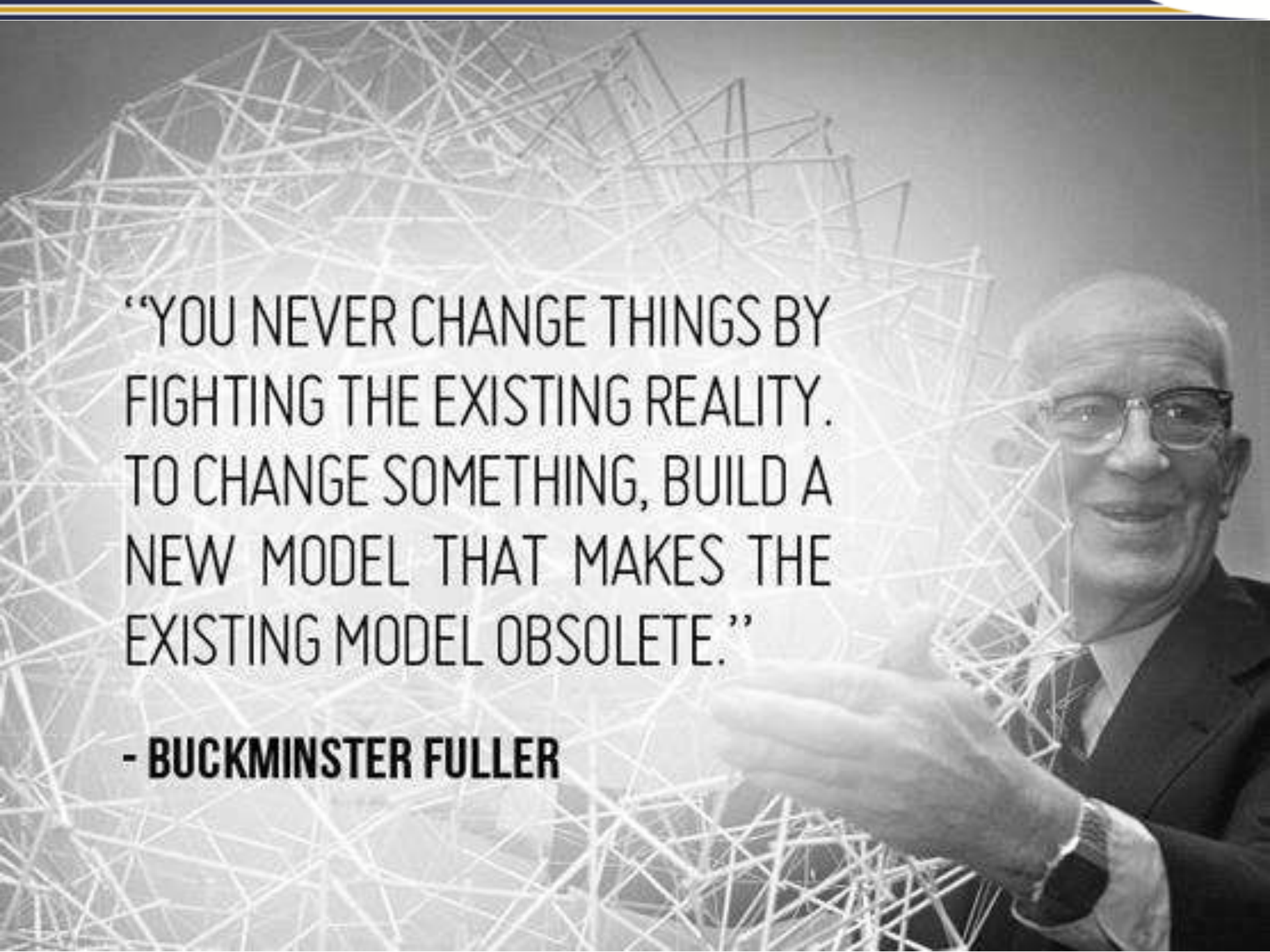


# Mathematics for the 21<sup>st</sup> Century



**Geneva May 2018**

**Charles@CurriculumRedesign.org**



“YOU NEVER CHANGE THINGS BY  
FIGHTING THE EXISTING REALITY.  
TO CHANGE SOMETHING, BUILD A  
NEW MODEL THAT MAKES THE  
EXISTING MODEL OBSOLETE.”

**- BUCKMINSTER FULLER**

# 1 Conference + 2 Colloquia with OECD



# Many thanks to

Sanjoy Mahajan



Zbigniew Marciniak



Bill Schmidt



Ralph Abraham – UC Santa Cruz



Michele Bruniges – New South Wales dept of education



Education Team - Alberta Education



Conrad Wolfram – Wolfram Research



# Report to download

## PISA Mathematics in 2021

An analysis of the [CENTER FOR CURRICULUM REDESIGN \(CCR\)](#)

### Authored by:

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Massachusetts  
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Zbigniew Marciniak



Bill Schmidt

MICHIGAN STATE  
UNIVERSITY

And Charles Fadel



### With many thanks to reviewers/contributors:

Ralph Abraham – UC Santa Cruz



Michele Bruniges – New South Wales dept of education



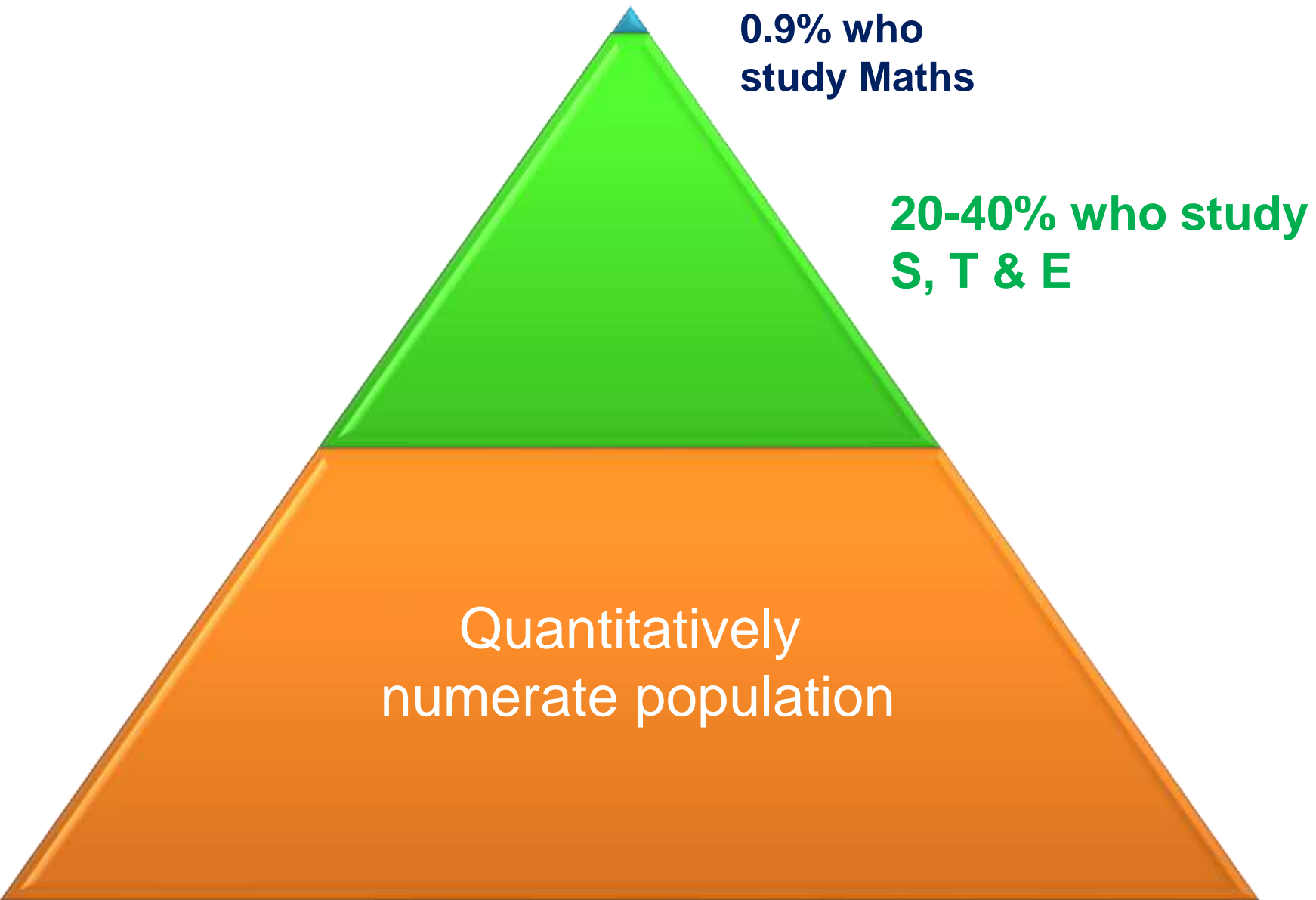
Education Team - Alberta Education



Conrad Wolfram – Wolfram Research

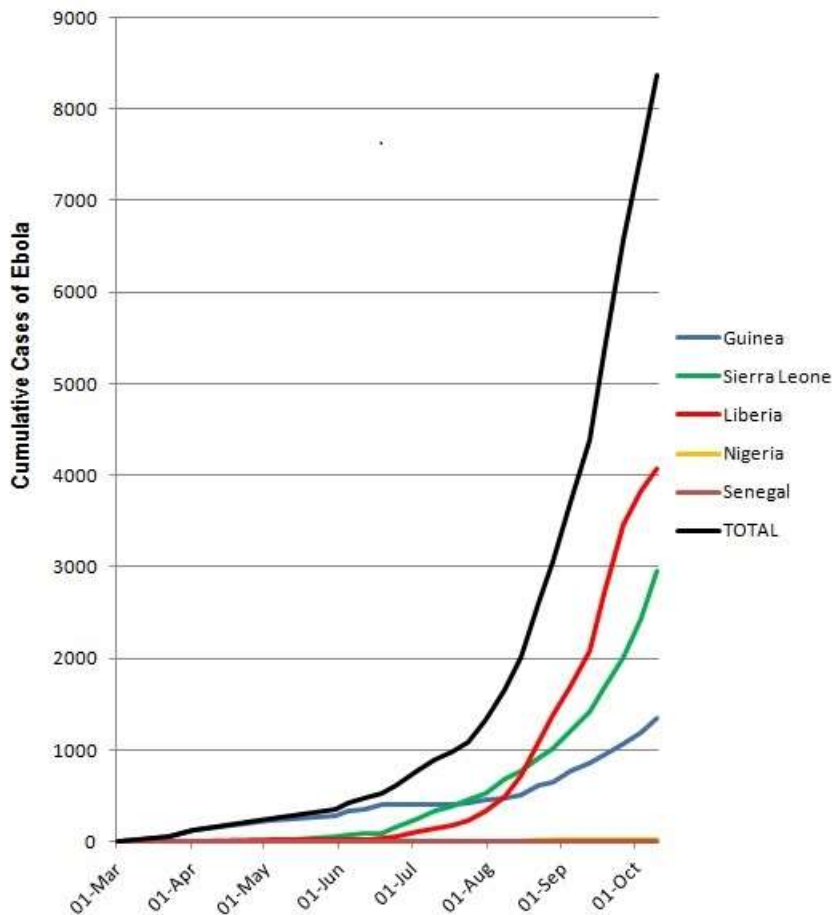


# Maths for *All Three Groups*



# Life needs deeper understanding of even *basic* Maths

Ebola in West Africa since March 2014

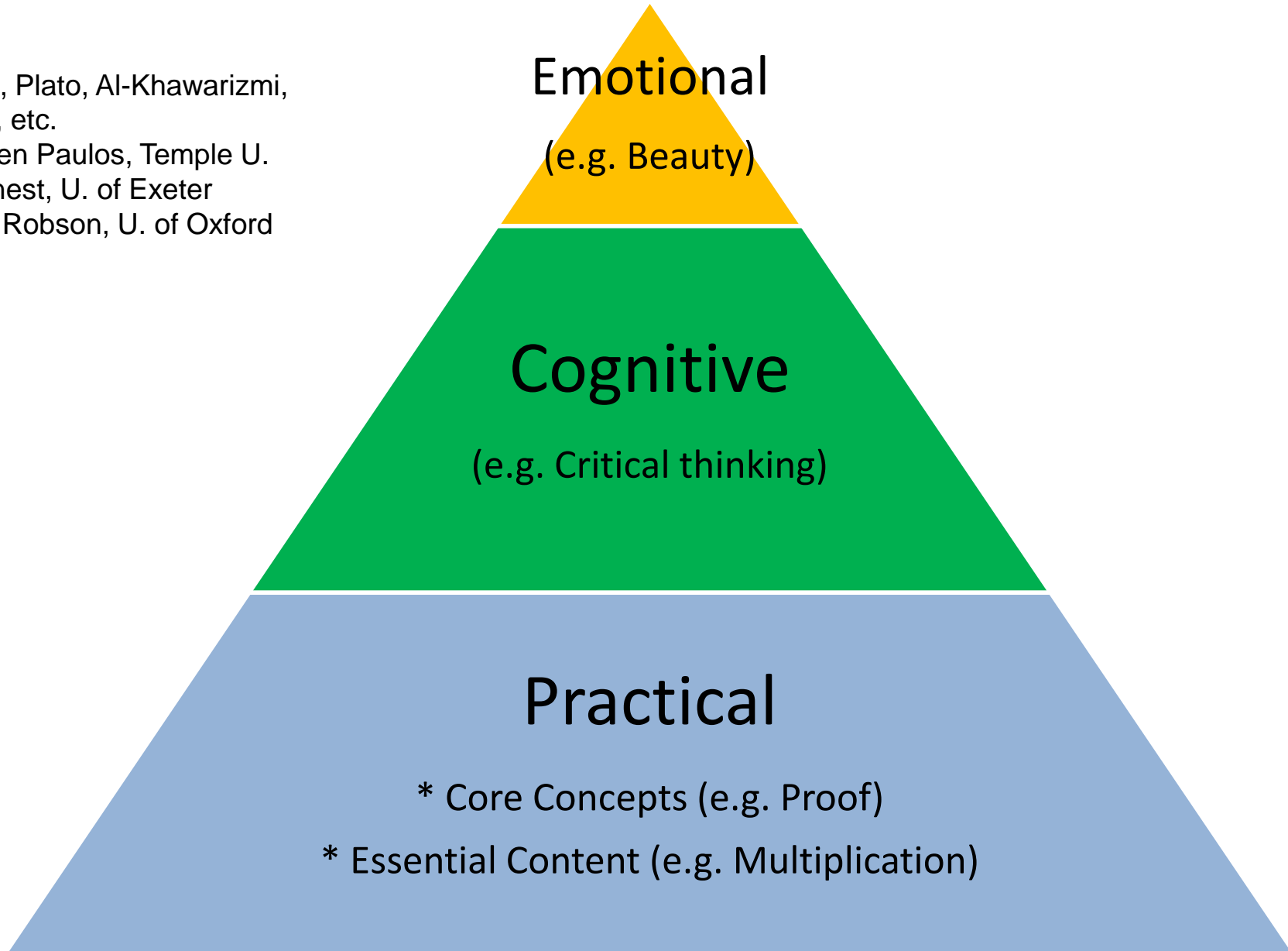


Principle of Proportionality

# What are the goals of learning Mathematics?

## Sources:

- Aristotle, Plato, Al-Khwarizmi, Al-Kindi, etc.
- John Allen Paulos, Temple U.
- Paul Ernest, U. of Exeter
- Eleanor Robson, U. of Oxford



**Emotional**

(e.g. Beauty)

**Cognitive**

(e.g. Critical thinking)

**Practical**

\* Core Concepts (e.g. Proof)

\* Essential Content (e.g. Multiplication)



# Modern industry needs different Maths

Themes	Responses
	<b>Knowledge</b>
Complexity	<b>Complex systems</b>
Uncertainty	<b>Statistics &amp; probabilities</b>
Multiple scales	<b>Complex systems</b>
Simulations & Modeling	<b>Computational Maths (algorithms)</b>
Data & Information	<b>Statistics &amp; probabilities</b>
	<b>Skills</b>
Multidisciplinarity	Collaboration
Transfer of knowledge	Communication

Source: OECD Global Science Forum Report on Mathematics in Industry

# Some observations

Most important in many occupations are

- Number, quantity, measure
- *Data handling and uncertainty*

Followed by

- Space and shape
- Relations, change, formulas

# Workplace Mathematics

- Mathematical modeling:
  - e.g. energy requirement of a water company; cost of sandwich; ...
- Use of Software, and coping with problems:
  - e.g. oil extraction; dispersion of sewage; ...
- Costing (allocation, dispute management):
  - e.g. Contract cleaning of hospital; management of railway; ...
- Performance and ratios:
  - e.g. Insurance ratios; glycemic index; ...
- Risk:
  - e.g. clinical governance; insurance; ...
- Quality/SPC control:
  - E.g. Furniture; machine downtime; deviation of rails; ...

# Relevance is a choice

© Charles Fadel Occupation (below)	Algebra	Applied Maths	Calculus	Discrete Mathematics	Foundations	Geometry	Numbers & Operations	Statistics & Probability	Topology & Recreational
Taxonomy & Ontology: Wolfram Research →	Matrices, Operations, Vectors etc	Complex systems, Control, Game theory, etc	Analysis, Transforms, Polynomials, etc	Automata, Graphs, Computational maths etc	Sets, Logic etc	Curves, Dimensions, Transformations, Trigonometry, etc	Arithmetic operations, Fractions, Sequences, etc	Distributions, Analysis, Estimation, etc	Knots, Figures, Folding, Spaces, etc
Agriculture						X	X	X	
Architecture		X				X	X	X	X
Astronomy/Cosmology	X	X	X	X		X	X	X	X
Biology, Botany, Zoology		X		X			X	X	
Biotechnology, Genetics	X	X	X	X		X	X	X	X
Business		X					X	X	
Cinematography/Photography						X	X		X
Civil engineering	X	X	X	X		X	X	X	X
Communication		X					X	X	
Computer science	X	X	X	X	X	X	X	X	X
Craftsmanship						X	X		X
Dance						X	X		X
Design						X	X		X
Drawing						X	X		X
Economics & Finance	X	X	X	X		X	X	X	
Education	X	X	X			X	X	X	
Electrical engineering	X	X	X	X		X	X	X	
Environmental science	X	X	X	X		X	X	X	
Ethics							X		
Geography/Geology	X	X	X	X		X	X	X	X
Health							X	X	
History/Archeology	X	X		X			X	X	
Journalism	X	X					X	X	
Languages/Linguistics	X	X		X			X	X	
Law		X					X	X	
Materials Science/Nanotechnology	X	X	X	X		X	X	X	X
Mechanical engineering, Robotics	X	X	X	X		X	X	X	X
Medicine/Pharmacy/Veterinary		X					X	X	
Music	X						X	X	
Painting						X	X		
Philosophy		X			X		X	X	
Physics	X	X	X	X	X	X	X	X	X
Poetry/Prose							X		
Psychology/Sociology/Anthropology	X	X		X			X	X	
Sculpture						X	X		X
Sewing/Knitting/Tapestry						X	X		X
Spirituality/Religions							X		
Theater/Acting							X		

# Change is Hard

From NSF report:

- “more emphasis on estimation, mental maths...
- “less emphasis on paper/pencil execution...”
- “content in... algebra, geometry, pre-calculus and trigonometry need to be... streamlined to make room for important new topics.”
- “discrete Mathematics, statistics/probabilities and computer science must be introduced”.

***Date of report: 1982 (!)***

# Why did we add Trigonometry in the 1800's?

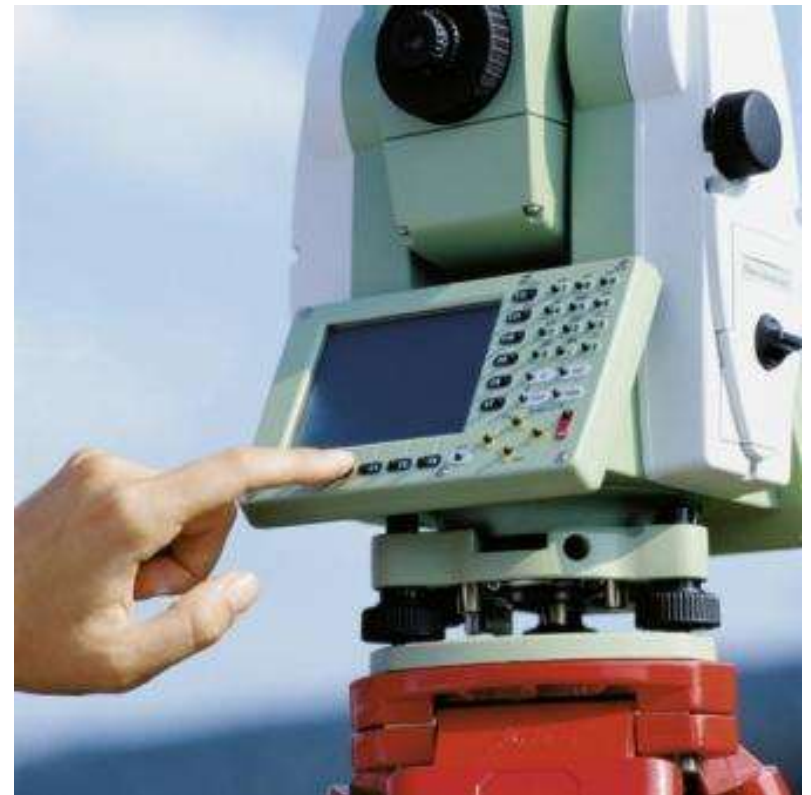
- a) Because it is pure
- b) Because the times required more land surveyors and woodworkers
- c) Because it was a predictor of college success

A: b) of course !

# Surveyor at Stanford U.

Me: “How do you use ArcSecant ?”

Him: “I don’t – it’s computerized”



# Wisdom from a Mathematician

*“Numbers and probability provide the basis for statistics, which, together with Logic, constitute the foundation of the Scientific Method”*

John Allen Paulos

Author, “A mathematician reads the newspaper”



# Wisdom from a Corporation

*“I keep saying the sexy job in the next ten years will be statisticians. People think I’m joking but who would’ve guessed that computer engineers would’ve been the sexy job of the 1990s.”*



Hal Varian, Google Chief Economist  
McKinsey Quarterly, Jan 2009

**Bloomberg  
Businessweek**

## **America’s Hottest Job – data scientist**

May 18, 2018

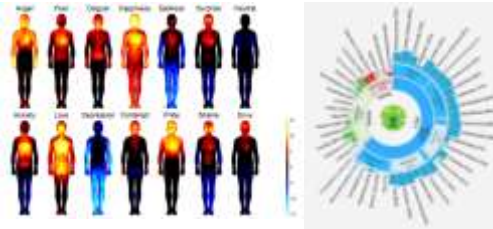
# Cognitive Domain and Algorithms

CREATING
Synthesizing
EVALUATING
ANALYZING
APPLYING
UNDERSTANDING
REMEMBERING



# Affective Domain and Algorithms

INTERNALIZING
ORGANIZING
VALUING
RESPONDING
RECEIVING



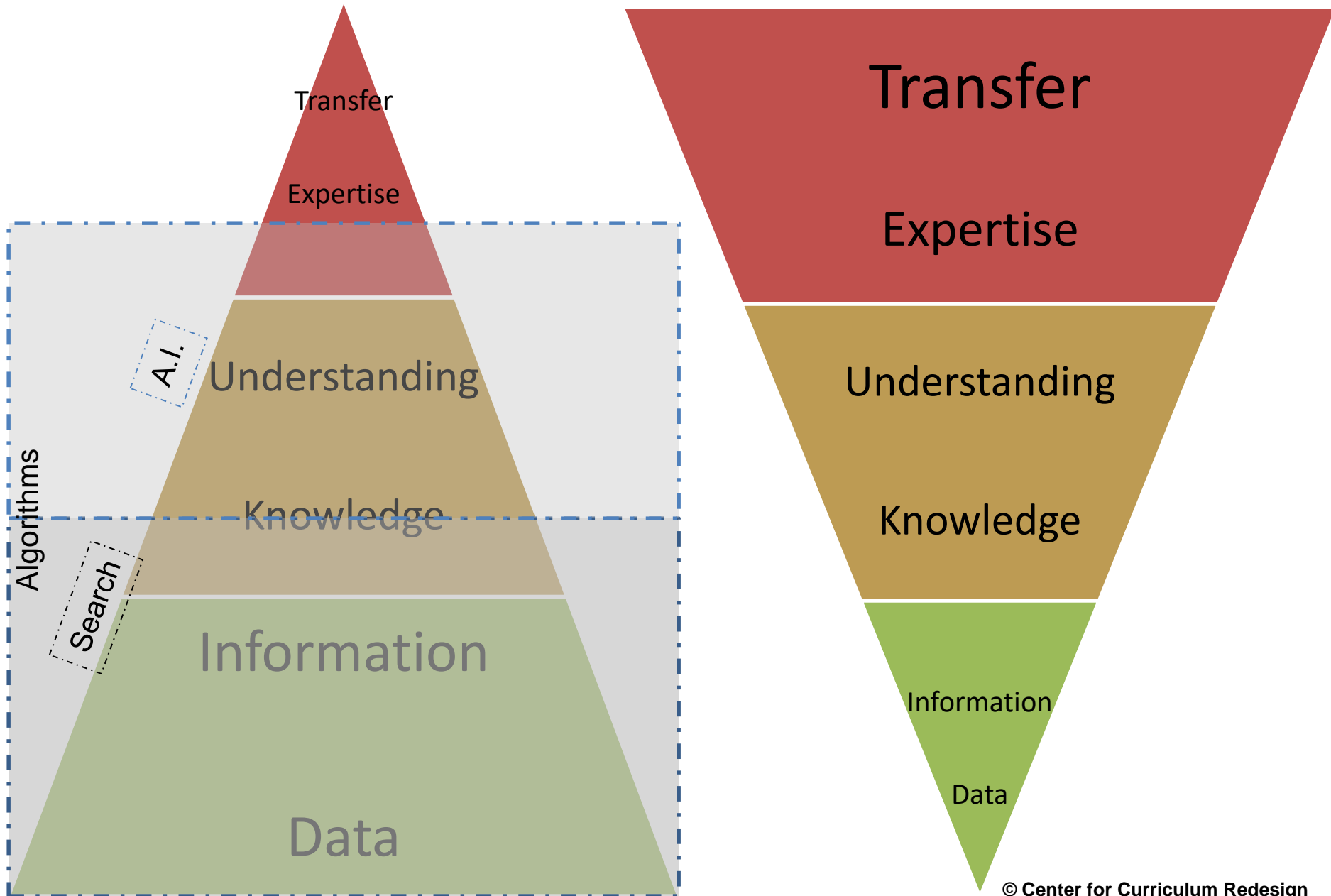
Source: Krathwohl, Bloom, Masia

# Psychomotor Domain and Algorithms

ORIGINATION
ADAPTATION
COMPLEX OVERT RESPONSE
MECHANISM
GUIDED RESPONSE
SET
PERCEPTION



# Flipping the Curriculum



# The Challenge – Expertise & Transfer

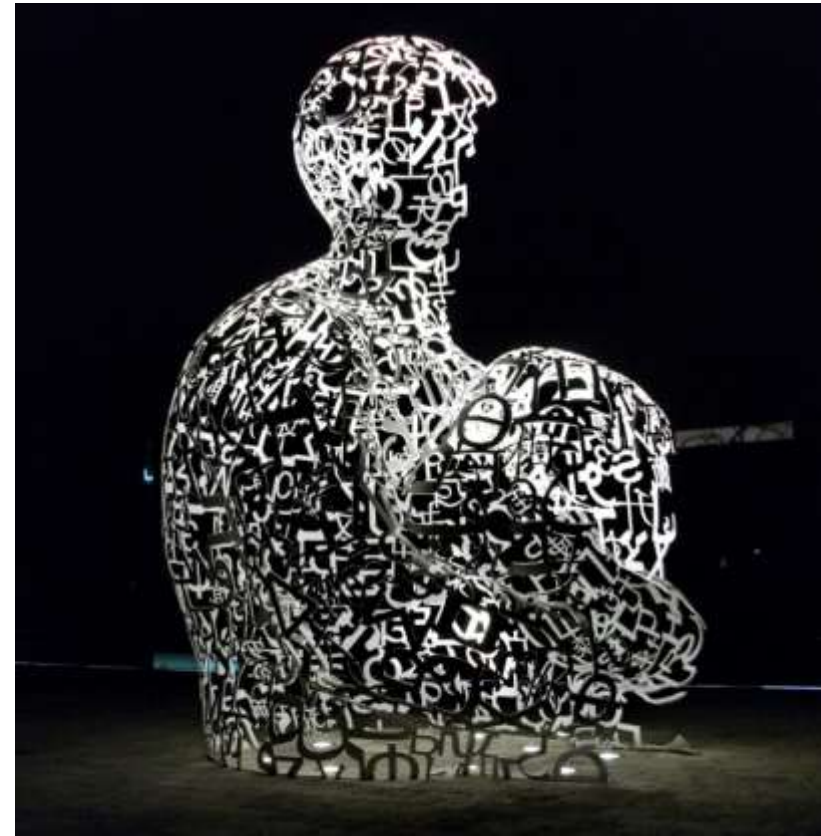
Save time and space for:

- More emphasis on *important* traditional areas
- Adding new, more *relevant* areas
- Deeper learning of core concepts
- Interdisciplinarity, for real-world relevance
- Developing Skills, Character, Meta-Learning

→ De-emphasize less relevant areas

# What should be Emphasized?

- Number sense & estimation
- Proportionality
- Exponentials
- Etc.



# What should be *added* ?

## Branches/topics:

- Recreational Maths
- Statistics & probabilities
- Discrete Maths
- Applied Maths

## Tools/Methods:

- Logic and argumentation
- Progression from concrete to abstract
- *Use of computer-based computation*
- *Linkages to the real-world*



# What should be *emphasized or added* ? (3)

## Core Concepts/Processes:

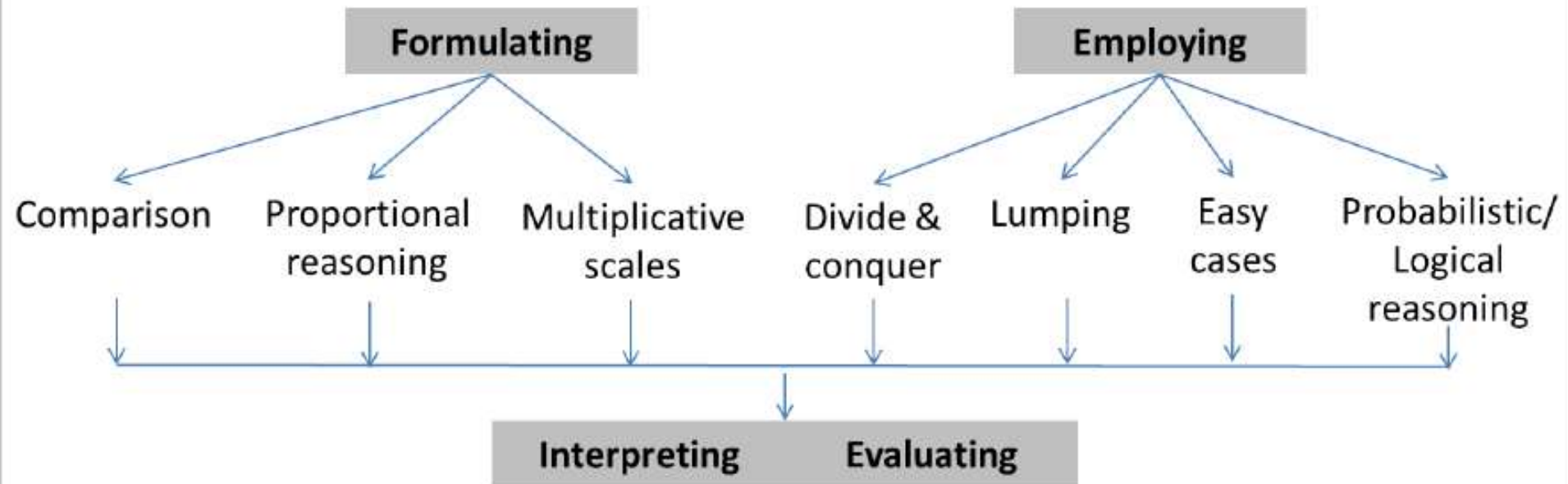
- Variable, rate, dimension, etc.
- Proving, representing, modelling etc.

## Other Dimensions of Education:

- Skills: Creativity, Critical thinking
- Character: Resilience
- Meta-Learning: Metacognition; Growth Mindset

# Critical Thinking: Explicit Reasoning

## Mathematical Processes



# Complete Matrix

## CCR Framework Matrix - DISCIPLINES

		Contribution welcome but not expected										
	x	Some contribution expected										
	X	Large contribution expected										

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## Competencies

Disciplines	Skills				Character						Meta-Learning	
	Creativity	Critical thinking	Communication	Collaboration	Mindfulness	Curiosity	Courage	Resilience	Ethics	Leadership	Growth Mindset	Metacognition
Mathematics	x	X	x	x		x		X			X	x

So...

*What do we remove ?*



→ Deep re-examination of every single branch, subject, topic, item...

...while fighting biases, groupthink, politics, etc.

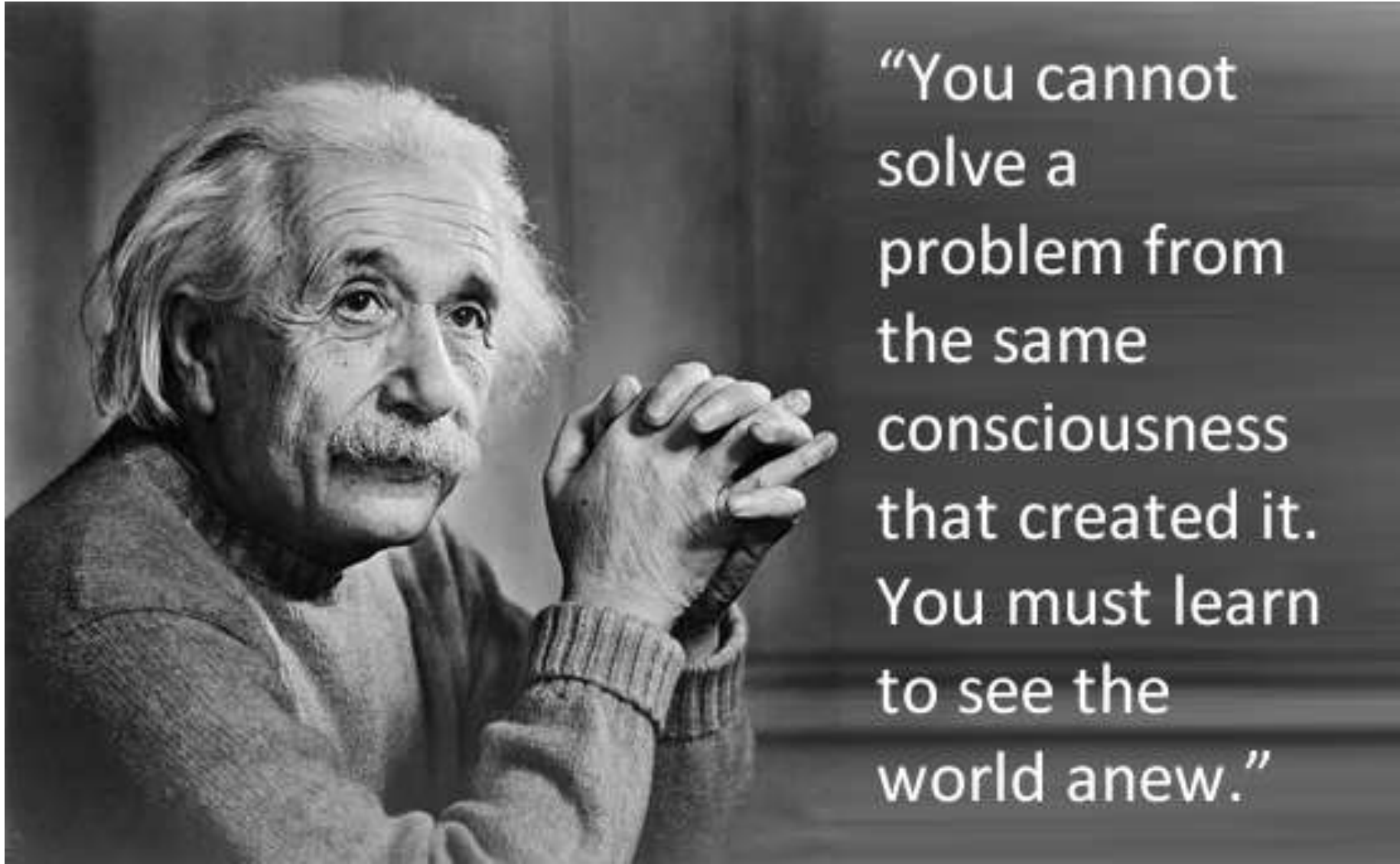
# Online Poll

Login: ISG-Visitor

Password: math21

<http://bit.ly/gvamaths>

# We agree



“You cannot solve a problem from the same consciousness that created it. You must learn to see the world anew.”

# What should be *de-emphasized* ?

## Branches/topics to carefully *curate*:

- Algebra
- Calculus
- Geometry/Trigonometry

## Tools/Methods:

By-hand algebraic computation

# Sobering Reality

*“...there is nothing... more dangerous to manage than a new system. For the initiator has the enmity of all who would profit by the preservation of the old institution and merely lukewarm defenders in those who gain by the new ones.”*

Niccolo Machiavelli  
“The Prince”





# Thank You !

“***What*** should students learn for the 21<sup>st</sup> century?”



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