Opinion Short Paper

Why are even ***basic*** *Probabilities so difficult to learn?*

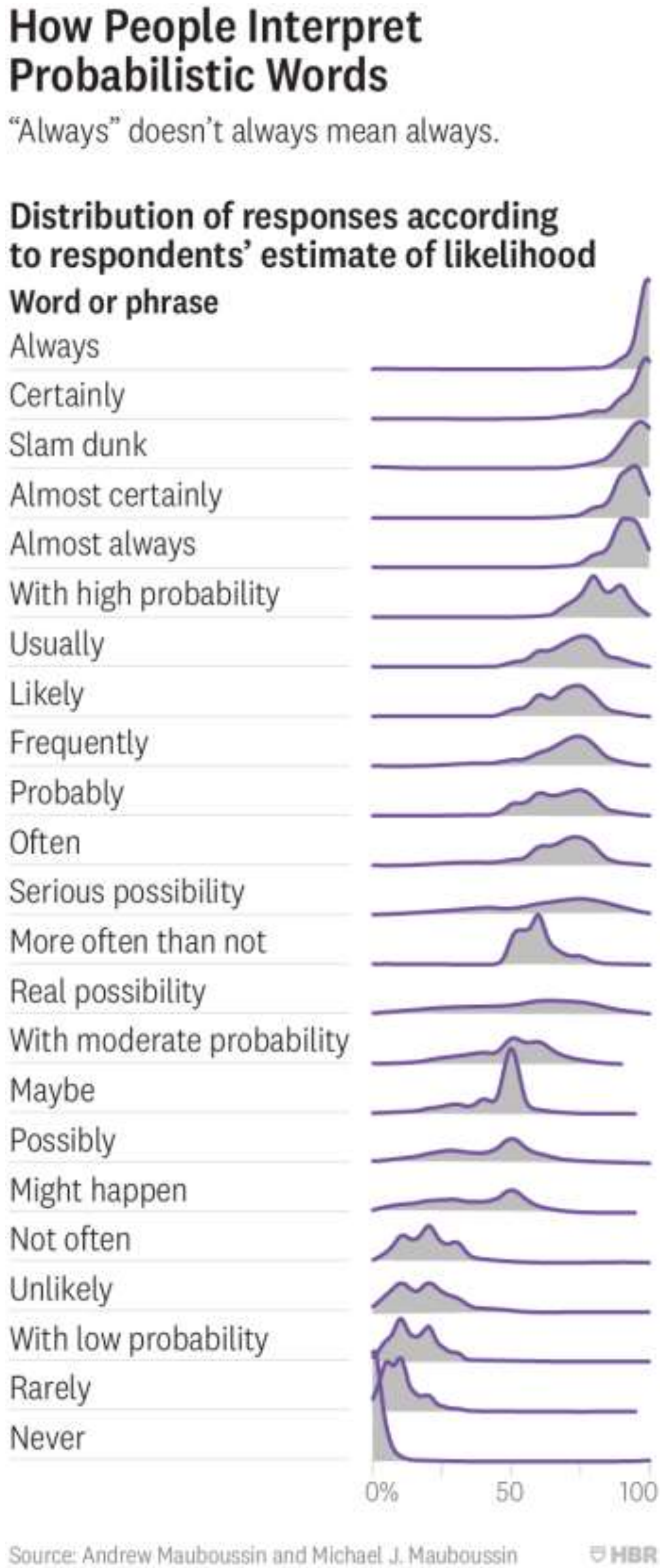
Probabilities are vital to survival, as many species are constantly evaluating the uncertainty surrounding them; evolution tweaks their behaviors to maximize their chances (e.g., school of fish diluting the probability of any one fish being eaten).

Yet this is one area of Mathematics that, surprisingly, teachers and students struggle to get right. We say "surprisingly", because the computational aspects are trivial: basic arithmetic is all that's needed! (and even factorials "!" are basic multiplications…).

In CCR's opinion, there are several reasons for this complexity:

1. Mathematically:
   a. The algebraic aspects are trivial, but the *logical reasoning* aspects can be convoluted: For example, in medicine, understanding false positives vs false negatives, and their computation, confuses even clinicians.
   b. Probabilities also rely on a deep understanding of Number Sense, which is insufficiently taught and relates to the psychology aspects (see below). For example: one chance in a million and one chance in a billion are nowhere near each other…
   c. Understanding of basic proportionality already causes problems for some: the proportion of doctors being males, and the proportion of males being doctors, is not at all the same.
   d. The undignified "religious wars" between Frequentists and Bayesians - both are right given their axioms/philosophy. But such "all or nothing" debates confuse students.
2. Linguistically:
   a. A small change in terminology has a huge impact: the example above on proportionality is telling.
   b. Words are imprecise, and using the wrong word can impede/confuse understanding: for example: preposterous vs possible vs plausible vs probable[1]; or colloquial use of faux-mathematical language that clouds people's understandings: "Odds are...", "I'll take my chances", "one in a million" and "inconceivable!"

[1] https://qz.com/1327180/heres-a-handy-tool-to-help-you-talk-about-probability/


How People Interpret Probabilistic Words
"Always" doesn't always mean always.
Distribution of responses according to respondents' estimate of likelihood
Word or phrase
Always
Certainly
Slam dunk
Almost certainly
Almost always
With high probability
Usually
Likely
Frequently
Probably
Often
Serious possibility
More often than not
Real possibility
With moderate probability
Maybe
Possibly
Might happen
Not often
Unlikely
With low probability
Rarely
Never
0%
50
100
Source: Andrew Mauboussin and Michael J. Mauboussin
HBR

3. Psychologically:
    a. Basic information literacy: humans are not good at expending the energy to verify sources - they trust the sender, if they are of the same tribe.
    b. Irrationality/Emotionality: Humans are not good[2] at thinking carefully, deliberately and logically. It is cognitively taxing, and we avoid it as much as possible. For instance, with Causality:
        i. Assignment of cause is complex, often multi-faceted and hard to track. Humans will try to find a fast shortcut.
        ii. “Hopeful causation”: when people say “Everything happens for a reason”: It answers the age-old metaphysical question “why do bad things happen to good people''. It helps cope with lack of control by offering hope[3].
    c. Unfairness: the sense of fairness is deeply rooted in mammals, as an evolutionary trait. We may be led astray in thinking a random result is “unfair” because of how it affects us.
    d. Randomness: Humans do not have a good intuitive sense of randomness; humans will generally select uniformly distributed plots as 'more random' than plots that were generated from a uniform distribution but therefore come out with apparent clusters[4].
    e. Variability: experiments run over a long period may have a surprisingly wide divergence in results[5], which is counter-intuitive to a “regression to the mean” mindset.
    f. Biases[6]: many from this list[7], and particularly:
        i. Confirmation bias: “favoring information that confirms or supports one's prior beliefs or values and choosing to ignore disconfirming evidence”. For example, not asking questions that would disconfirm your theory, such as in this activity.

[2] D. Kahneman “Thinking fast & slow”. D. Ariely “Predictably irrational”.
[3] “when the Lord closes a door, He opens a window” – in the movie The Sound of Music
[4] The human skew towards lower integers (Benford's Law) is not mentioned here as it applies only to data that ranges over several orders of magnitude, like the prices of all one’s purchases in one year, which might include 2 dollars and 2000 dollars. Also, it refers to the leading digit, and only occurs in a few contexts.
[5] From Pr. David Aldous, UC Berkeley: “a deck of cards with the S&P 500 gains for each of the last 52 years, like +23%, - 12%, etc is simulated for the next 10 years by dealing 10 cards; that gives (say) an overall 86% gain. But repeating will give surprisingly different answers -- 22% or 122%, for instance.”
[6] Definition in quotations are from Wikipedia
[7] https://en.wikipedia.org/wiki/List_of_cognitive_biases#Prospect_theory

  ii. Anchoring bias: "relying too heavily on the first piece of information given about a topic". For example, if you see a pen for $500 and then one for $80, you are going to think the $80 pen is cheap.
  iii. Negativity bias: registering negative stimuli more readily than positive ones and dwelling on these events. For example, when you get feedback on a presentation and focus on the one thing you could do better instead of on the five pieces of feedback which were positive.
  iv. Prospect theory: "placing more of an emphasis on potential gains than potential losses". This leads to sunk cost behavior: continuing a faulty system by using past investment as a rationale. For example, if a stock is doing poorly, investors may be reluctant to sell it because then the money they've already put in is officially unrecoverable.
  v. Fundamental Attribution Error: "under-emphasizing situational and environmental explanations for an individual's observed behavior, while over-emphasizing dispositional and personality-based explanations".[8] For example, when a colleague makes a mistake, you are likely to interpret it as related to a flaw they have and less likely to consider that they happened to not get enough sleep that day.
  vi. Representativeness heuristic: Expressed by people as "if I can't think of an example, it must be rare; and conversely, if I can, it must be common."
  vii. Gambler's fallacy: people believe short sequences of random events should be representative of longer ones, such as the occurrence of heads vs tails coin tosses after a long series of one of them.

g. Decision-making:
  i. Follow-through: Understanding a situation is one thing, dealing with it is another: it may require the Courage to face it and not succumb to denialism.
  ii. Hope is a powerful motivator: even in low-odds situations, such as poor people buying lottery tickets.

[8] https://en.wikipedia.org/wiki/Fundamental_attribution_error

iii. Impact: The probability of an event is not sufficient. What matters is *probability x impact*; if you have a 1% chance of melting down the financial system of the world, shouldn't you be paying attention?[9]

h. The philosophical aspects were puzzling even to Pascal: how can the language of precision (mathematics) describe the imprecise?

Probabilities require understanding all of the above, not just the (relatively simple) computational aspects.

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[9] Nassim Nicholas Taleb: "The Black Swan"